QB365 Question Bank Software Study Materials

Differential Calculus - Limits and Continuity Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Complete the table using calculator and use the result to estimate the limit.

$lim_{x\to2}\frac{x-2}{x^2-4}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	0.25641	0.25062	0.250062	0.24993	0.24937	0.24390

Answer : Let $f(x)=\frac{x-2}{x^2-4}=\frac{x-2}{(x-2)(x+2)}=\frac{1}{x+2}$

$\therefore \lim_{x\to2}\frac{x-2}{x^2-4}=\lim_{x\to2}\frac{1}{x+2}=\frac{1}{4}=0.25$

2) In problem, using the table estimate the value of the limit

$lim_{x\to-3}\frac{\sqrt{1-x}-2}{x+3}$

x	-3.1	-3.01	-3.00	-2.999	-2.99	-2.9
f(x)	– 0.24845	– 0.24984	– 0.24998	– 0.25001	– 0.25015	– 0.25158

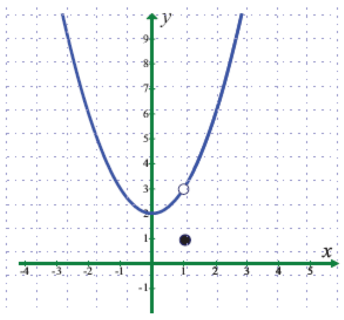
Answer : Let $f(x)=\frac{\sqrt{1-x}-2}{x+3}$

$\therefore \lim_{x\to-3}\frac{\sqrt{1-x}-2}{x+3}=-0.250$

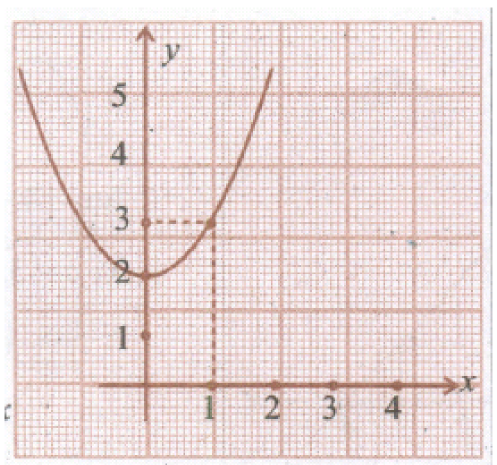
3) Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$lim_{x\to1}f(x)$

where $f(x)=\begin{cases}x^2+2, & x\neq1\\ 1, & x=1\end{cases}$.



Answer :

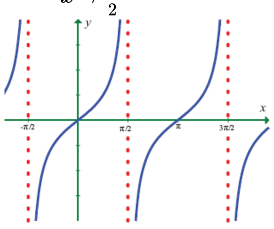


At x = 1, the value of the curve on the y-axis is 3.

$\therefore lim_{x\to1}f(x)=3$

4) Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$lim_{x\to\frac{\pi}{2}}tanx$



Answer : tan x can be made arbitrarily large when x is chosen suficiently close to $\frac{\pi}{2}$ in the left side.

Similarly it can be made arbitrarily small when x is chosen sufficiently closer to $\frac{\pi}{2}$ in the right side.

x is chosen sufficiently closer $\frac{\pi}{2}$ to in the right side.

x is chosen sufficiently closer $\frac{\pi}{2}$ to in the right side.

tan x does not approach any value when x approaches $\frac{\pi}{2}$

$Indeed \lim_{x \to \frac{\pi}{2}^-} \tan x = and \lim_{x \to \frac{\pi}{2}^+} \tan x = -\infty.$

Hence the limit does not exist.

5) If the limit of f(x) as x approaches 2 is 4, can you conclude anything about f(2)? Explain reasoning.

Answer : Limit of f(x) as x approaches 2 is the nature of (x) on both sides of 2.

It is independent ofthe nature of f(x) at x = 2.

Therefore we can not conclude anything about $lim_{x \to 2} f(x) from f(2) = 4$

6) Calculate $lim_{x \to x_o}(5)$ for any real number x_0.

Answer : f(x) = 5 is a polynomial (of degree 0).

Hence $lim_{x \to x_o}(5) = f(x_o) = 5$.

7) Compute $lim_{x \to 1} \frac{x^3-1}{x-1}$

Answer : $lim_{x \to 1} \frac{x^3-1}{x-1} = lim_{x \to 1} \frac{x^3-1}{x-1} = 3(1)^{3-1} = 3.$

8) Evaluate the following limits :

$lim_{x \to 2} \frac{x^4-16}{x-2}$

Answer : $lim_{x \to 2} \frac{x^4-16}{x-2} = lim_{x \to 2} \frac{x^4-2^4}{x-2}$ $[\because lim_{x \to a} \frac{x^n-a^n}{x-a} = n.a^{n-1}]$

= $4(2)^{4-1} = 4(2)^3 = 4(8) = 32$

9) Evaluate the following limits :

$lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$

Answer : $lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$

Multiplying and dividing by $\sqrt{x+4}+3$ we get,

$lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5} \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$ = $lim_{x \to 5} \frac{(x+4)-3^2}{x-5[\sqrt{x+4}+3]}$ = $lim_{x \to 5} \frac{(x+4)-9}{(x-5)[\sqrt{x+4}+3]}$

$= \lim_{x \to 5} \frac{x-5}{(x-5)\left[\sqrt{x+4}+3\right]} = \lim_{x \to 5} \frac{1}{\sqrt{x+4}+3}$

$= \frac{1}{\sqrt{5+4}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$

10) Evaluate the following limits :

$lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

Answer : $lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

Multiplying and dividing by $(\sqrt{1+x}+1)$ we get,

$lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = lim_{x \to 0} \frac{(1+x)-1}{x[\sqrt{1+x}+1]}$

$= \lim_{x \to 0} \frac{x}{x\left[\sqrt{1+x}+1\right]}$

$= lim_{x \to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2}$

11) Evaluate the following limits :

$lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

Answer : $lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

Multiplying and dividing by $(\sqrt{x-1}+2)$ we get,

$lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5} \times \frac{\sqrt{x-1}+2}{\sqrt{x-1}+2} = lim_{x \to 5} \frac{(x-1)-4}{x-5[\sqrt{x-1}+2]}$

$= \lim_{x \to 5} \frac{(x-5)}{(x-5)\left[\sqrt{x-1}+2\right]}$

$= lim_{x \to 5} \frac{1}{\sqrt{x-1}+2} = \frac{1}{\sqrt{5-1}+2}$

$\frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

$lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5} = \frac{1}{4}$

12) Calculate $lim_{x \to \infty} \frac{x^3+2x+3}{(5x^2+1)}$.

Answer : Dividing by x^2

$lim_{x\to\infty}\frac{x^3+2x+3}{(5x^2+1)} = lim_{x\to\infty}\frac{x+\frac{2}{x}+\frac{3}{x^2}}{5+\frac{1}{x^2}} \to \infty$

That is, $\frac{x^3+2x+3}{(5x^2+1)} \to \infty\ as x \to \infty.$

In other words, the limit does not exist.

Note that the degree of numerator is higher than that of the denominator.

13) Calculate $lim_{x\to\infty}\frac{1-x^3}{3x+2}$

Answer : Dividing by x, we get

$\frac{1-x^3}{3x+2} = \frac{\frac{1}{x}-x^2}{3+\frac{2}{x}} \to -\infty\ as x \to \infty$

Therefore the limit does not exist.

14) Evaluate the following limits :$lim_{x\to\infty}(1+\frac{1}{x})^{7x}$

Answer : $lim_{x\to\infty}(1+\frac{1}{x})^{7x}= lim_{x\to\infty}[(1+\frac{1}{x})^{x}]^7$

Put $\frac{1}{x}=t,$

when $x\to\infty$ means $\frac{1}{x}\to 0$

$\therefore \frac{1}{x}\to 0$ means $t\to 0$

$=[lim_{t\to 0}[(1+t)^{\frac{1}{t}}]^7 = e^7\ [\because lim_{x\to 0}(1+x)^{\frac{1}{x}} = e]$

$\therefore lim_{x\to\infty}(1+\frac{1}{x})^{7x} = e^7$

15) Prove that f(x) = $2x^2$ + 3x - 5 is continuous at all points in **R**.

Answer : Given f(x) = $2x^2$ + 3x - 5

f(x) is an algebraic function.

Since the algebraic function is continous in **R**

f(x) is comtinous at all points in **R**.

16) Examine the continuity of the following : $\frac{x^2-16}{x+4}$

Answer : Let f(x) $=\frac{x^2-16}{x+4}$

The algebraic function is continuous for all $x\in R.$

Since the curve f(x) does not exist for x = -4, the given function is continuous only in R-{-4}.

17) Examine the continuity of the following:cot x + tan x

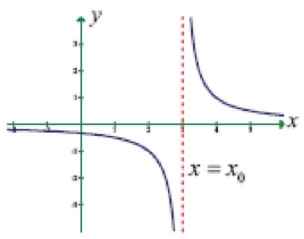
Answer : Let f(x) = cot x + tan x

cot x is not continuous in multiples of π and tan x is not continuous in (2n+1)$\frac{\pi}{2}$.

$\therefore$ f(x) = cot x + tan x is not continuous in (2n+1)$\frac{\pi}{2}$+$\frac{\pi}{2}$,

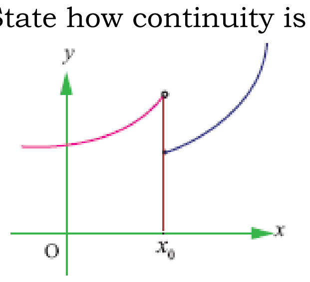
$\Rightarrow$ f(x) is continuous in R $-\frac{n\pi}{2}, n\in Z.$

18) State how continuity is destroyed at x = x_o for each of the following graphs.



Answer : The limit of f(x) does not exist at x = $x_{o.}$

19) State how continuity is destroyed at x= x_o for each of the following graphs.



Answer : The left-hand limit and right-hand limit does not coincide at x=x_o.

20) Find $\lim_{x\to 0}\frac{(2+x)^5-2^5}{x}$

Answer : Put 2 + x = y so that as y → 2 as x → 0.

Therefore, $\lim_{x\to 0}\frac{(2+x)^5-2^5}{x}=\lim_{y\to 2}\frac{y^5-2^5}{y-2}=5\left(2^4\right)=80$

21) Evaluate $\lim_{x\to 1}\frac{1+(x-1)^2}{1+x^2}$

Answer : $\lim_{x\to 1}\frac{1+(x-1)^2}{1+x^2}=\frac{1+0}{1+1^2}=\frac{1}{2}$

22) Evaluate$\lim_{x\to 0}\frac{x^{\frac{2}{3}}-9}{x-27}$

Answer : $\lim_{x\to 0}\frac{x^{\frac{2}{3}}-9}{x-27}=\lim_{x\to 0}\frac{x^{\frac{2}{3}}-(27)^{\frac{2}{3}}}{x-27}=\frac{2}{3}(27)^{\frac{2}{3}-1}=\frac{2}{3}(27)^{\frac{-1}{3}}$

$=\frac{2}{3(27)^{\frac{1}{3}}}=\frac{2}{3(3^3)^{\frac{1}{3}}}=\frac{2}{3(3)}=\frac{2}{9}$

23) Evaluate $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-x-2}$

Answer : $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-x-2}=\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x+1)}=\lim_{x\to 2}\frac{(x-1)}{(x+1)}=\frac{2-1}{2+1}=\frac{1}{3}$

24) Evaluate: $\lim\limits_{x\to 0}\frac{e^{5x}-1}{x}$

Answer : $\lim\limits_{5x\to 0}\frac{e^{5x}-1}{x}\times 5=5(1)$

25) If $f(x)=\{\begin{array}{ll}2x+3, & x\le 0\\ 3(x+1), & x>0\end{array}$ .Find $\lim\limits_{x\to 0}\lim\limits_{x\to 1}f(x)$ and $\lim\limits_{x\to 1}f(x)$

Answer : $f(x)=\{\begin{array}{ll}2x+3, & x\le 0\\ 3(x+1), & x>0\end{array}$

$\therefore \lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^-}(2x+3)=2\times 0+3=3$

and $\lim\limits_{x\to o^+}f(x)=\lim\limits_{x\to 1^-}3(x+1)=3(0+1)=3$

So,$\lim\limits_{x\to 0}f(x)$ exists and is equal to 3.

$\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}2x+3=2\times 1+3=5$

$\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}2x+3=2\times 1+3=5$

$\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}3(x+1)=3(1+1)=6$

$\lim\limits_{x\to 1^-}f(x)\neq\lim\limits_{x\to 1^+}f(x)$

Hence,$\lim\limits_{x\to 1}f(x)$ does not exist.

26) Find $\lim\limits_{x\to 1}$ fix), if $f(x)=\{\begin{array}{ll}x^2-1 & x\le 1\\ -x^2-1 & x>1\end{array}$

Answer : We have,

$f(x)=\{\begin{array}{ll}x^2-1 & x\le 1\\ -x^2-1 & x>1\end{array}$

$\therefore \lim\limits_{x\to 1^-}=\lim\limits_{x\to 1^-}x^2-1=1^2-1=0$

and $\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}-x^2-1=-1-1=-2$

$\therefore \lim\limits_{x\to 1}f(x)\neq\lim\limits_{x\to 1^+}f(x)$

Hence,$\lim\limits_{x\to 1}f(x)$ does not exist

27) Evaluate $\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2}$

Answer : We have,

$\lim\limits_{n\to\infty}\frac{n!}{(n+1)!-n!}=\lim\limits_{n\to\infty}\frac{n!}{(n+1)\,n!-n!}=\lim\limits_{n\to\infty}\frac{1}{n+1-1}$

$=\lim\limits_{n\to\infty}\frac{1}{n}=0$

28) Find $\lim_{x\to 3}\frac{x^3-6x^2+9x}{x^2-9}$

Answer : Write

$F(x)=x^3-6x^2+9x=x(x-3)^2=(x-3)f(x)$

where $f(x)=x(x-3)$

$G(x)=x^2-9=(x-3)(x+3)$

$=(x-3)g(x)$ where $g(x)=x+3$

$\therefore \frac{F(x)}{G(x)}=\frac{(x-3)f(x)}{(x-3)g(x)}=\frac{f(x)}{g(x)}$ and $g(3)=6\neq 0$

Now, by applying Theorem we get,

$\lim_{x\to 3}\frac{x^3-6x^2+9x}{x^2-9}=\lim_{x\to 3}\frac{F(x)}{G(x)}=\frac{f(3)}{g(3)}=\frac{3(3-3)}{3+3}=\frac{0}{6}=0$

29) Compute $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

Answer : We have $\lim_{x\to 0}\frac{e^{3x}-1}{x}=3\lim_{3x\to 0}\frac{e^{3x}-1}{3x}$

$=3\lim_{y\to 0}\frac{e^{y}-1}{y}$, where $Y=3x$

$=3\cdot 1=3$

30) If $f:R\to R$ is such that $f(x+y)=f(x)+f(y)$ for allix, $y\in R$, then f is continuous on ' R if it is continuous at a single po

Answer : Let f be continuous at $x_0\in R$.

Then $\lim_{t\to x_0}f(t)=f(x_0)$

$\lim_{h\to 0}f(x_0+h)=f(x_0)$

Let $x\in R$. Now, since

$(f(x+h)-f(x))=f(x_0+h)-f(x_0)$ we have

$\lim_{h\to 0}(f(x_0+h)-f(x_0))=0$

Therefore f is continuous at x.

Since $x\in R$ is arbitrary, f is continuous on R.