QB365 Question Bank Software Study Materials

Integral Calculus Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Integrate the following with respect to x : $\sqrt{x}$

Answer : $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}x^{\frac{3}{2}} + c$

2) Integrate the following with respect to x : $\frac{1}{\sqrt{x}}$

Answer : $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = 2\sqrt{x} + c$

3) Integrate the following with respect to x : $\frac{sin\ x}{cos^2\ x}$

Answer : $\int \frac{sin\ x}{cos^2\ x}dx = \int \frac{sin\ x}{cos\ x}.\frac{1}{cos\ x}\ dx = \int tan\ x sec\ x\ dx\ = sec\ x + c$

4) Integrate the following with respect to x : $\frac{1}{x^7}$

Answer : $\int \frac{1}{x^7}dx = \frac{x^{-7+1}}{-7+1} + c = \frac{x^{-6}}{-6} + c = -\frac{1}{6x^6} + c$

5) Integrate the following with respect to x : $\sqrt[3]{x^4}$

Answer : $\int \sqrt[3]{x^4}dx = \int \left(x^4\right)^{1/3}dx$

$= \int x^{\frac{4}{3}}dx$

$= \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + c = \frac{3}{7}x^{\frac{7}{3}} + c$

6) Integrate the following with respect to x : $\left(x^5\right)^{\frac{1}{8}}$

Answer : $\int \left(x^5\right)^{1/8}dx = \int x^{\frac{5}{8}}dx$

$= \frac{x^{\frac{5}{8}+1}}{\frac{5}{8}+1} + c = \frac{8}{13}x^{\frac{13}{8}} + c$

7) Integrate the following with respect to x : $\frac{1}{sin^2 x}$

Answer : $\int \frac{1}{sin^2 x}dx = \int cosec^2 x dx$

= − cot x +c

8) Integrate the following with respect to x : $\frac{tan\ x}{cos\ x}$

Answer : $\int \frac{\tan x}{\cos x} = \int \sec x \tan x dx$

$= \sec x + c$

9) Integrate the following with respect to x : $\left(1 - x^2\right)^{-\frac{1}{2}}$

Answer : $\int \left(1 - x^2\right)^{-\frac{1}{2}}dx = \int \frac{1}{\left(1-x^2\right)^{1/2}}dx$

$= \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + c$

10) IIntegrate the following functions with respect to x : $(x + 5)^6$

Answer : $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int (x + 5)^6 dx = \frac{(x+5)^{6+1}}{6+1} + c$

= $\frac{(x+5)^7}{7} + c$

11) Integrate the following functions with respect to x : sin 3x

Answer : $\int \sin x dx = -\cos x + c$

$\therefore \int \sin 3x dx = \frac{-\cos 3x}{3} + c$

12) Integrate the following with respect to x : $(1-x^3)^2$

Answer : $\int (1-x^3)^2 dx = \int (1-2x^3+x^6) dx$

$= \int dx - 2\int x^3\ dx + \int x^6\ dx$

$= x - \frac{x^4}{2} + \frac{x^7}{7} + c.$

13) Integrate the following with respect to x : $e^{3x}(e^{2x}-1)$

Answer : $\int e^{3x}(e^{2x}-1)dx = \int (e^{5x}-e^{3x})dx = \frac{e^{5x}}{5} - \frac{e^{3x}}{3} + c.$

14) Evaluate : $\int \sqrt{1+\cos 2x} dx$

Answer : $\int \sqrt{1+\cos 2x} dx = \int \sqrt{2\cos^2 x} dx = \sqrt{2}\int \cos x dx = \sqrt{2}\sin x + c$

15) Evaluate :$\int \sqrt{1+sin\ 2x} dx$

Answer : $\int \sqrt{1+sin\ 2x} dx = \int \sqrt{(cos^2\ x + sin^2\ x) + (2sin\ xcos\ x)} dx$

$= \int \sqrt{(cos^2\ x + sin\ x)^2} dx = \int (cos\ x + sin\ x) dx$

= sin x - cos x + c

16) Integrate the following functions with respect to x : cos3x cos2x

Answer : $\cos(mx)\cos(nx) = \frac{\cos(mx+nx)+\cos(mx-nx)}{2}$

$= \int \cos 3x \cos 2x dx$

$= \int \frac{1}{2}[\cos(3x-2x) + \cos(3x+2x)]dx$

$= 1/2[[\cos x + \cos 5x]dx$

$= \frac{1}{2}\left[\sin x + \frac{\sin 5x}{5}\right] + c$

17) Integrate the following functions with respect to x : $e^{xloga} e^x$

Answer : $\int e^{z\log a} \times e^x dx = \int e^{\log a^2} \times e^x dx$

$= \int a^x \times e^x dx \quad (\because e^{\log u} = u)$

$= \int (ae)^x dx$

$= \frac{(ae)^x}{\log ae} + c$

Aliter :

$\int e^{x\log a} \times e^x dx = \int e^{x\log a + x} dx$

$= \int e^{x(\log a+1)} dx$

$= \frac{e^{\log a+1)x}}{(\log a+1)} + c$

18) Integrate the following with respect to x : $\frac{x^2}{1+x^6}$

Answer : Let $I = \int \frac{x^2}{1+x^6} dx$

put t = $x^3 \Rightarrow dt = 3x^2 dx$

$\frac{dt}{3} = x^2 dx$

$I = \int \frac{dt}{1+t^2}$

$= \frac{1}{3}\int \frac{dt}{1+t^2}$

$= \frac{1}{3}\tan^{-1}(t) + c$

$= \frac{1}{3\tan^{-1}}(x^3) + c$

19) Evaluate the following integrals : $\int e^{3x} cos\ 2x\ dx$

Answer : $\int e^{3x} cos\ 2x\ dx$

Using the formula

$\int e^{ax} cos\ bx\ dx = \frac{e^{ax}}{a^2+b^2}[a\ cos\ bx + b\ sin\ bx] + c$

For a = 3 and b = 2, we get

$\int e^{3x} cos\ 2x\ dx = (\frac{e^3 x}{3^2+2^2})(3cos\ 2x + 2sin\ 2x) + c$

$= (\frac{e^{3x}}{13})(3\ cos\ 2x + 2\ sin 2x) + c$

20) Evaluate the following integrals : $\int e^{-5x} sin 3x\ dx$

Answer : $\int e^{-5x} sin3x\ dx$

Using the formula

$\int e^{ax} sin\ bx\ dx = \frac{e^{ax}}{a^2+b^2}[a\ sin\ bx + b\ cos\ bx] + c$

for a = -5, b = 3, we get

$\int e^{-5x} sin\ 3x\ dx = (\frac{e^{-5x}}{(-5)^2+3^2})(-5sin3x - 3cos3x) + c$

$\int e^{-5x} sin\ 3x\ dx = -(\frac{e^{-5x}}{34})(5sin3x - 3cos3x) + c$

21) Evaluate : $\int \left(\frac{x^4+1}{x^2+1}\right)$ dx

Answer : $\therefore \frac{x^4+1}{x^2+1}$ = x^2 - 1 + $\frac{2}{x^2+1}$

$\therefore \int \frac{x^4+1}{x^2+1}$ dx = $\int \left(x^2 - 1\right) dx + 2\int \frac{dx}{x^2+1}$

= $\frac{x^3}{3}$ - x + 2$\tan^{-1}$ x + c

22) Evaluate : $\int \frac{cosxdx}{(1+sinx)(2+sinx)}$dx

Answer : Let I = $\int \frac{cosxdx}{(1+sinx)(2+sinx)}$

Put t = sin x ⇒ dt = cos x dx

$\therefore$ I = $\int \frac{dt}{(1+t)(2+t)}$

Now, $\frac{1}{(1+t)(2+t)}$ = $\frac{A}{1+t} + \frac{B}{2+t} = \frac{A(2+t)(1+t)}{(1+t)(2+t)}$

1 = A (2 + t) + B (1 + t)

When t = -1 1 = A (1) ⇒ A = 1

When t = -2 1 = B (-1) ⇒ B = -1

$\therefore \frac{1}{(1+t)(2+t)}$ = $\frac{1}{1+t} - \frac{1}{2+t}$

I = $\int \frac{dt}{(1+t)(2+t)} = \int \frac{1}{1+t}dt - \int \frac{1}{2+t}dt$

= log |1 +t| - log |2+t| + c = $log\left|\frac{1+t}{2+t}\right|$+c

I = $log\left|\frac{1+sinx}{2+sinx}\right| + c$ [$\therefore$ t = sin x]

23) Integrate the function with respect to x : $\frac{1}{x^5}$

Answer : $\int \frac{1}{x^5}dx = \int x^{-5}dx = \frac{x^{-5+1}}{-5+1} = \frac{x^{-4}}{-4} = \frac{-1}{4x^4} + c$

24) Solve : cos(4x + 5)

Answer : $\int cos\,(4x+5)\,dx = \frac{sin(4x+5)}{4} = \frac{1}{4}sin\,(4x+5) + c$

25) Integrate the function with respect to x : $4 - \frac{5}{x+2} + 3cos2x$

Answer : $\int \left[4 - \frac{5}{x+2} + 3cos2x\right] dx$

= $4\int dx - 5\int \frac{1}{x+2}dx + 3\int cos2xdx$

= $4x - 5log(x+2) + \frac{3}{2}sin2x + c$

26) Integrate the function with respect to x : x(l - x)^{16}

Answer : = ∫x (l - x)^{16}

put t = I - x ~ x = 1- t; dt = -dx, dx = -dt

$I = \int (1-t)\,t^{16}(-dt) = \int -lt^{16} + t^{17}dt = \int \left(t^{17} - it^{16}\right) dt$

= $\frac{t^{18}}{18} - \frac{lt^{17}}{17} = \frac{(1-x)^{18}}{18} - \frac{l(l-x)^{17}}{17} + c$

27) Evaluate : $\left(sin^{-1}x\right) \frac{e^{sin^{-1}x}}{\sqrt{1-x^2}}$

Answer : $\int sin^{-1}x\dfrac{e^{sin^{-1}x}}{\sqrt{1-x^2}}dx$

Let $t = sin^{-1}; dt = \dfrac{1}{\sqrt{1-x^2}}dx$

u = t; du = dt

e'dt = dv v = e'

$I = \int te'dt = \int td\,(e') = \int e^{'}dt = te^{'} - e' = e'(t-1)$

$e^{sin-1}x\left\{sin^{-1}x - 1\right\} + c$

28) Integrate the function with respect to x : $\sqrt{(2x+1)^2+9}$

Answer : = $\int\sqrt{(2x+1)^2+9}dx = \int\sqrt{(2x+1)^2+3^2}\,dx$

= $\dfrac{\frac{1}{2}\{2x+1\}\sqrt{(2x+1)^2+3^2}+\frac{9}{2}log(2x+1)+\sqrt{(2x+1)^2+9}}{2}$

= $\dfrac{1}{4}\{2x+1\}\sqrt{(2x+1)^2+9\}}+9log\{2x+1\}+\sqrt{(2x+1)^2\}}+c$

29) Find $\int(1-x)(4-3x)(3+2x)dx, x \in R.$

Answer : $\int(1-x)\,(4-3'x)\,(3+2x)dx = \int\left(4-7x+3x^2\right)(3+2x)dx$

$= \int\left(6x^3-5x^2-13x+12\right)dx$

$= 6\int x^3dx - 5\int x^2dx - 13\int xdx + 12\int dx$

$= \frac{3}{2}\dot{x}^4 - \frac{5}{3}x^3 - \frac{13}{2}x^2 + 12x + c$

30) Evaluate : $\int\cos^3 x\sin xdx$ on R.

Answer : $\int\cos^3 x\sin xdx$

Take $t = \cos x$

$dt = -\sin xdx$

$\Rightarrow \sin xdx = -dt$

$\int\cos^3 x\sin xdx = -\int t^3dt$

$= -\frac{t^4}{4} + c$

$= -\frac{\cos^4 x}{4} + c$