QB365 Question Bank Software Study Materials

Introduction To Probability Theory Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Can two events be mutually exclusive and independent simultaneously?

Answer : If A and B are mutually exclusive, then

$P(A\cap B)=0$

But if A and B are independent, then

$P(A\cap B)=P(A)\cdot P(B)$

So if A and B are non empty, then they are not mutually exclusive and independent simultaneously.

2) If P(A) = 0.5, P(B) = 0.8 and P(B/A) = 0.8, find P(A/B) and P(A∪B)

Answer : Given P(A) = 0.5, P(B) = 0.8

⇒ P(B/A) = 0.8

We kmow P(B/A) = $\frac{P(A\cap B)}{P(A)}$

⇒ 0.8 = $\frac{P(A\cap B)}{0.5}$

⇒ $P(A\cap B)=(0.8)(0.5)=0.4$

(i) Now P(A./B) = $\frac{P(A\cap B)}{P(B)}=\frac{0.4}{0.8}=\frac{1}{2}=0.5$

(ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

= 0.5 + 0.8 - 0.4 = 1.3 - 0.4

= 0.9

3) If A and B are two independent events such that P(A∪B) = 0.6, P(A) = 0.2, find P(B).

Answer : Given A and B are independent

$\Rightarrow P(A\cap B)=P(A)P(B)$

$P(A\cup B)=0.6, P(A)=0.2$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)\cdot P(B)$

$0\cdot 6=0\cdot 2+P(B)(1-0\cdot 2)$

$\frac{0\cdot 4}{0\cdot 8}=P(B)$

i.e., $P(B)=\frac{1}{2}=0.5$

4) If for two events A and B, P(A) = $\frac{3}{4}$, P(B) = $\frac{2}{5}$ and A∪B = S (sample space), find the conditional probability P(A/B).

Answer : $P(A)=\frac{3}{4}, P(B)=\frac{2}{5}, A\cup B=S$

$\Rightarrow P(A\cup B)=1$

$P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\frac{3}{4}+\frac{2}{5}-1$

$=\frac{15+8}{20}-1=\frac{23-20}{20}=\frac{3}{20}$

$P(A/B)=\frac{P(A\cap B)}{P(B)}$

$=\frac{3/20}{2/5}=\frac{3}{8}$

5) If an experiment has exactly the three possible mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probability is permissible.

$P(A)=\frac{4}{7}, P(B)=\frac{1}{7}, P(C)=\frac{2}{7}$

Answer : Since the experiment has exactly the three possible mutually exclusive outcomes A, B and C, they must be exhaustive events.

$\Rightarrow S=A\cup B\cup C$

Therefore, by axioms of probability

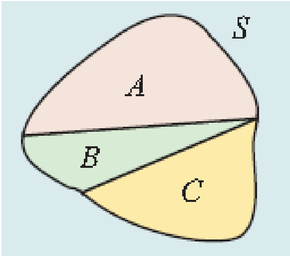
$P(A) \geq 0, P(B) \geq P(C) \geq 0$ and

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(S) = 1$

Given that $P(A) = \frac{4}{7} \geq 0, \quad P(B) = \frac{1}{7} \geq 0, \quad P(C) = \frac{2}{7} \geq 0$

Also $P(S) = P(A) + P(B) + P(C) = \frac{4}{7} + \frac{1}{7} + \frac{2}{7} = 1$

Therefore the assignment of probability is permissible



6) If an experiment has exactly the three possible mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probability is permissible

$P(A) = \frac{2}{5}, \quad P(B) = \frac{1}{5}, \quad P(C) = \frac{3}{5}$

Answer : Since the experiment has exactly the three possible mutually exclusive outcomes A, B and C, they must be exhaustive events.

$\Rightarrow S = A \cup B \cup C$

Therefore, by axioms of probability

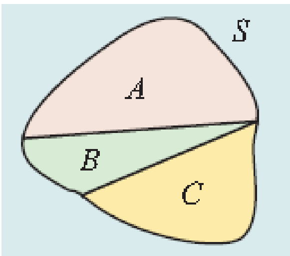
$P(A) \geq 0, P(B) \geq P(C) \geq 0$ and

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(S) = 1$

Given that $P(A) = \frac{2}{5} \geq, \quad P(B) = \frac{1}{5} \geq 0, \quad P(C) = \frac{3}{5} \geq 0$

But $P(S) = P(A) + P(B) + P(C) = \frac{2}{5} + \frac{1}{5} + \frac{6}{5} > 1$

Therefore the assignment is not permissible



7) If an experiment has exactly the three possible mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probability is permissible.

$P(A) = 0.3, P(B) = 0.9, P(C) = -0.2$

Answer : Since the experiment has exactly the three possible mutually exclusive outcomes A, B and C, they must be exhaustive events.

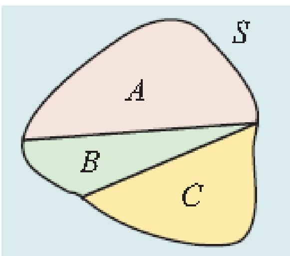
$\Rightarrow S = A \cup B \cup C$

Therefore, by axioms of probability

$P(A) \geq 0, P(B) \geq P(C) \geq 0$ and

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(S) = 1$

Since 0.2 P (C) = −0.2 is negative, the assignment is not permissible



8) If an experiment has exactly the three possible mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probability is permissible.

$P(A) = \frac{1}{\sqrt{3}}, \quad P(B) - 1 - \frac{1}{\sqrt{3}}, \quad P(C) - 0$

Answer : since the experiment has exactly the three possible mutually exclusive outcomes A, B and C, they must be exhaustive events.

$\Rightarrow S = A \cup B \cup C$

Therefore, by axioms of probability

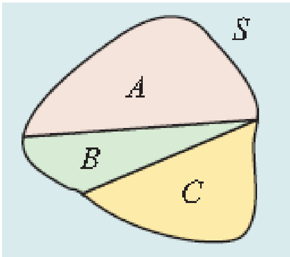
$P(A) \geq 0, P(B) \geq P(C) \geq 0$ and

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(S) = 1$

The assignment is permissible because

$P(A) = \frac{1}{\sqrt{3}} \geq 0, \quad P(B) - 1 - \frac{1}{\sqrt{3}} \geq 0, \quad P(C) - 0 \geq 0$

$P(S) = P(A) + P(B) + P(C) = \frac{1}{\sqrt{3}} + 1 - \frac{1}{\sqrt{3}} + 0 - 1$



9) If an experiment has exactly the three possible mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probability is permissible.

P(A) = 0.421, P(B) = 0.527 P(C) = 0.042

Answer : since the experiment has exactly the three possible mutually exclusive outcomes A, B and C, they must be exhaustive events.

$\Rightarrow S = A \cup B \cup C$

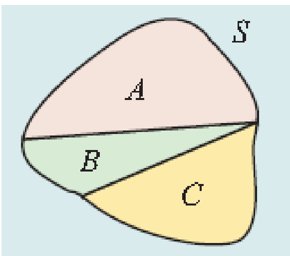
Therefore, by axioms of probability

$P(A) \geq 0, P(B) \geq P(C) \geq 0$ and

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(S) = 1$

Even though P(A) + P(B) + P(C) = 0.421 + 0.527 + 0.042 = 0.990 < 1

therefore, the assignment is not permissible



10) If P(A) = 0.6, P(B) = 0.5, and P(A∩B) = 0.2. Find P(A/B)

Answer : Given that P(A) = 0.6, P(B) = 0.5, and P(A∩B)=0.2

P(A/B) =$\frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$

11) If two coins are tossed simultaneously, then find the probability of getting (i) one head and one tail (ii) at most two tails

Answer : The sample space is S = {HH, HT, TH, TT}

⇒ n(S) = 4

(i) Let A be the event of getting one head and one tail, then

A = {HT, TH}

n(A) = 2

$\therefore \; P(A) = \frac{n(B)}{n(S)} = \frac{4}{4} = 1$

(ii) Let A be the event of getting atmose two tails, then

∴ B = {HH,HT,TH,TT}

$\therefore \; P(B) = \frac{n(B)}{n(S)} = \frac{4}{4} = 1$

12) A single card is drawn from a pack of 52 cards. What is the probability that The card is an ace or a king?

Answer : S = {Pack of 52 cards}

∴ n(S) = 52

P(ace card or a king card) = P(ace card) + P(king card)

= $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

13) A single card is drawn from a pack of 52 cards. What is the probability that The card will be 6 or smaller?

Answer : P(card will be 6 or smaller)

= $\frac{5+5+5+5}{52} = \frac{20}{52} = \frac{5}{13}$ [∵ 5 cards which are 6 or smaller from each variety]

14) A single card is drawn from a pack of 52 cards. What is the probability that The card is either a queen or 9?

Answer : (iii) P(queen card or 9)=$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

15) (i) The odds that the event A occurs is 5 to 7, find P(A)..

(ii) Suppose $P(B) = \frac{2}{5}$, Express the odds that the event B occurs.

Answer : $(i) a = 5, b = 7, P(A) = \frac{a}{a+b} = \frac{5}{5+7} = \frac{5}{1 \cdot 2}$

$(ii) P(B) = \frac{2}{5} = \frac{a}{a+b}$

$a = 2, a + b = 5$

$b = 3$

The odds that the event B occurs is 2 to 3.

16) Find the probability of getting the number 7, when a usual die is rolled.

Answer : The event of getting 7 is an impossible event. Therefore, P(getting 7) = 0

17) Nine coins are tossed once, find the probability to get at least two heads

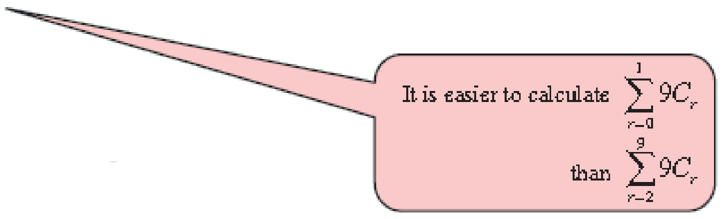
Answer : Let S be the sample space and A be the event of getting at least two heads.

Therefore, the event Ā denotes, getting at most one head.

n(S) = 2^9 = 512, n(Ā) = $9C_0$ + $9C_1$= 1 + 9 = 10

$P(\bar{A}) = \frac{10}{512} = \frac{5}{256}$

$P(A) = 1 - P(\bar{A}) = 1 - \frac{5}{256} = \frac{251}{256}$



18) If $P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.2$ Find $P(A/\bar{B})$

Answer : $P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$

$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$

$= \frac{0.6 - 0.2}{1 - 0.5} = \frac{0.4}{0.5} = \frac{4}{5}$

19) Given that P(A) =0.52, P(B)=0.43, and P(A∩B)=0.24, find

P$\left(\overline{A} \cap \overline{B}\right)$

Answer : P$\left(\overline{A} \cap \overline{B}\right)$=$P(\overline{A \cup B})$ (By de morgan's law)

=1-P(A∪B)

=1-0.71=0.29

20) Given that P(A) = 0.52, P(B) = 0.43, and P(A∩B) = 0.24, find $P(A \cup B)$

Answer : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.52 + 0.43 - 0.24$

$P(A \cup B) = 0.71$

21) Events A and B are such that P(A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B are independent?

Answer : Given P(A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P($\bar{A}\cup\bar{B}$) = $\frac{1}{4}$.

Now, $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B}) = 1 - P(A\cap B)$

$\Rightarrow \frac{1}{4} = 1 - P(A\cap B) \quad \Rightarrow P(A\cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

Now P(A) × P(B) = $\frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

$\therefore P(A\cap B) \neq P(A)\times P(B)$

Thus, A and B are not independent.

22) Given that the events A and B are such that P(A) = $\frac{1}{2}$, P(AUB) = $\frac{3}{5}$ and P(B) = p. find P if they are mutually exclusive events.

Answer : Since A and B are mutually exclusive, P(A⋂B) = 0

Now, P(AUB) = P(A) + P(B) - P(A⋂B)

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0$

$\Rightarrow p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$

$\therefore p = \frac{1}{10}$

23) The ratio of the number of boys to the number of girls in a class is 1:2. It is known that the probability of a girl and a boy getting a first class are 0.25 and 0.28 respectively. Find the probability that a student chosen at random will get first class?

Answer : Let E_1 and E_2 be the events of choosing a boy and a girl respectively from the class.

Given that the number of boys to the number of girls = 1: 2

$\therefore P(E_1) = \frac{1}{1+2} = \frac{1}{3}$ and P(E_2) = $\frac{2}{1+12} = \frac{2}{3}$

Let A be the event that a student chosen will get first class

Given P(A/E_1) = 0.28 and P(A/E_2) = 0.25

∴ By theorem of total probability,

P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2)

$\Rightarrow \quad = \frac{1}{3}\times 0.28 + \frac{2}{3}\times 0.25$

$= \frac{28}{300} + \frac{50}{300} = \frac{78}{300} = 0.26$

24) An experiment has the four possible mutually exclusive outcomes A, B, C and D, Check whether the following assignments of probability are permissible.

p(A) = 0.32, P(B) = 0.28, P(C) = - 0.06, P(D) = 0.46

Answer : Probability of an event cannot be negative. Here P(C) =-0.06

∴ the above set of events are not possible.

P(A) =$\frac{1}{3}$, P(B)=$\frac{1}{6}$, P(C) =$\frac{2}{9}$, P(D)=$\frac{5}{18}$

25) Two cards are drawn one by one at random from a deck of 52 playing cards. What is the probability of getting two jacks if

(i) the first card is replaced before the second card is drawn

(ii) the first card is not replaced before the second card is draw?

Answer : n(S) = 52;$P(J) = \frac{4}{52}$

(i) $P(JJ) = \frac{4}{52}\times\frac{4}{52} = \frac{1}{13}\times\frac{1}{13} = \frac{1}{169}$

(ii) $P(JJ) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{13}\times\frac{1}{17} = \frac{1}{221}$

26) A number x is drawn arbitrarily from the set {1, 2, 3,....., 100} flind the probability that $\left[x + \frac{100}{x}\right] > 29$

Answer : The total,points of the sample are 100. Let A be the event that an x selected (drawn) at random from the set s= {1, 2, 3,....., 100} has the property $\left[x + \frac{100}{x}\right] > 29$

Now $x + \frac{100}{x} > 29$

$\Rightarrow x^2 - 29x + 100 > 0$

$\Rightarrow (x-4)(x-25) > 0$

$\Rightarrow x > 25$ (or) $x < 4$

Since $x \in S$, it follows that

$A = \{1,2,3,26,27,\ldots,100\}$. Thus the number of cases favorable to A is 78.

$\therefore$ The required probability: $P(A) = \frac{78}{100} = 0.78$

27) Two squares are chosen at random on a chess board. Show that the.probability that they have a side in common is $\frac{1}{18}$

Answer : The number of ways choosing the first Square is 64 and that of the second is 63.

Therefore the number of ways of choosing the first and the second Square is 64 x 63.

Let E be the event that these squares have a side in common. We shall find the number of cases favorabl e to E.

If the first square happens to be one of the squares in the four corners of the chessboard, the second square (with cornmon side) can be chosen in 2 ways.

If the first square happens to be any one of the remaining 24 squares alone the four sides of the chess board other than the corner,the second square can be chosen in 3 ways.

If the first square happens to be any one of the remaining 36 inner squares, then the second square can be chosen in 4 ways.

Hence the number of cases favorable to E is

(4 x 2) + (24 x 3) + (36 x 4) = 224

Therefore the required probability is $\frac{224}{64\times63}=\frac{1}{18}$

28) A fair coin is tossed 200 times. Find the probability of getting a head an odd number of times

Answer : The total number of points in the sample is 2^{200} Let E be event of geuing a head an odd numbers of times. Then the numbers of cases favorable to E is

$^{200}C_1+{}^{200}C_3+\ldots+{}^{200}C_{199}=\frac{2^{200}}{2}=2^{199}$

$\therefore$ The required probability $P(E)=\frac{2^{199}}{2^{200}}=\frac{1}{2}$

29) A and B are among 20 persons who sit at random along a round table. Find the probability that there are any six persons between A and B

Answer : Let A occupy any seat at the round table. Then thgre are 19 seats left for B. But if six person are to be seated between A and B, then .B has only two ways to sit. Thus the required probability is $\frac{1}{19}$

30) Out of 30 consecutive integers. Two integers are drawn at random. Find the probability that their sum is odd.

Answer : The total nirmber of ways of choosing 2 integers out of 30 is $^{30}C_2$. Out of the 30 numbers, 15 are even and 15 are odd. If the sum.of the tvio numbers is to be odd, one should be even and ttre other odd. Hence the number of cases favourable to the required event is $^{15}C_1\times{}^{15}C_1$

$\therefore$ The required probability $=\frac{^{15}C_1\times{}^{15}C_1}{^{30}C_2}$

$=\frac{15\times15\times2}{30\times29}=\frac{15}{29}$