QB365 Question Bank Software Study Materials

Matrices and Determinants Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Suppose that a matrix has 12 elements. What are the possible orders it can have? What if it has 7 elements?

Answer : The number of elements is the product of number of rows and number of columns.

Therefore, we will find all ordered pairs of natural numbers whose product is 12.

Thus, all the possible orders of the matrix are 1×12, 12×1, 2×6, 6×2, 3×4 and 4×3.

Since 7 is prime, the only possible orders of the matrix are 1×7 and 7×1.

2) Find the sum A + B + C if A, B, C are given by

$$A=\begin{bmatrix}\sin^2\theta & 1\\ \cot^2\theta & 0\end{bmatrix}, B=\begin{bmatrix}\cos^2\theta & 0\\ -\operatorname{cosec}^2\theta & 1\end{bmatrix} \text{ and } C=\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$

Answer : By the definition of sum of matrices, we have

$$A+B+C=\begin{bmatrix}\sin^2\theta+\cos^2\theta+0 & 1+0-1\\ \cot^2\theta-\operatorname{cosec}^2\theta-1 & 0+1+0\end{bmatrix}=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$$

3) Determine 3B + 4C - D if B, C, and D are given by

$$B=\begin{bmatrix}2 & 3 & 0\\ 1 & -1 & 5\end{bmatrix}, C=\begin{bmatrix}-1 & -2 & 3\\ -1 & 0 & 2\end{bmatrix}, D=\begin{bmatrix}0 & 4 & -1\\ 5 & 6 & -5\end{bmatrix}$$

Answer : $3B+4C-D=\begin{bmatrix}6 & 9 & 0\\ 3 & -3 & 15\end{bmatrix}+\begin{bmatrix}-4 & -8 & 12\\ -4 & 0 & 8\end{bmatrix}+\begin{bmatrix}0 & -4 & 1\\ -5 & -6 & 5\end{bmatrix}=\begin{bmatrix}2 & -3 & 13\\ -6 & -9 & 28\end{bmatrix}$.

4) Simplify : $sec\theta\begin{bmatrix}sec\theta & tan\theta\\ tan\theta & sec\theta\end{bmatrix}-tan\theta\begin{bmatrix}tan\theta & sec\theta\\ sec\theta & tan\theta\end{bmatrix}$

Answer : If we denote the given expression by A, then using the scalar multiplication rule, we get

$$A=\begin{bmatrix}sec^2\theta & sec\theta tan\theta\\ sec\theta tan\theta & sec^2\theta\end{bmatrix}-\begin{bmatrix}tan^2\theta & tan\theta sec\theta\\ sec\theta tan\theta & tan^2\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

5) Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

$a_{ij}=\frac{(i-2j)^2}{2} with\ m=2, n=3$

Answer : Given $a_{ij}=\frac{(i-2j)^2}{2} with\ m=2, n=3$

We need to construct a 2×3 matrix.

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix}$$

$a_{11}=\frac{(1-2(1))^2}{2}=\frac{(-1)^2}{2}=\frac{1}{2}$

$a_{12}=\frac{(1-2(2))^2}{2}=\frac{(-3)^2}{2}=\frac{9}{2}$

$a_{13}=\frac{(1-2(3))^2}{2}=\frac{(-5)^2}{2}=\frac{25}{2}$

$a_{21}=\frac{(2-2(1))^2}{2}=\frac{0}{2}=0$

$a_{22}=\frac{(2-2(2))^2}{2}=\frac{(-2)^2}{2}=\frac{4}{2}$

$a_{23}=\frac{(2-2(3))^2}{2}=\frac{(-4)^2}{2}=\frac{16}{2}$

$\therefore A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{pmatrix}=\begin{pmatrix}1/2 & 9/2 & 25/2\\ 0 & 4/2 & 16/2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1 & 9 & 25\\ 0 & 4 & 16\end{pmatrix}$

6) Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

$a_{ij}=\frac{|3i-4j|}{4} with\ m=3, n=4$

Answer : Let B be a 3×4 matrix with entries as

$B=\begin{pmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\end{pmatrix}$

$$\begin{pmatrix} a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$

$a_{ij} = \frac{|3i-4j|}{4}$

$a_{11} = \frac{|3-4|}{4} = \frac{|-1|}{4} = \frac{1}{4}$

$a_{12} = \frac{|3-8|}{4} = \frac{|-5|}{4} = \frac{5}{4}$

$a_{13} = \frac{|3-12|}{4} = \frac{|-9|}{4} = \frac{9}{4}$

$a_{14} = \frac{|3-16|}{4} = \frac{|-13|}{4} = \frac{13}{4}$

$a_{21} = \frac{|3(2)-(4)1|}{4} = \frac{|6-4|}{4} = \frac{2}{4}$

$a_{22} = \frac{|3(2)-4(2)|}{4} = \frac{|6-8|}{4} = \frac{2}{4}$

$a_{23} = \frac{|3(2)-4(3)|}{4} = \frac{|6-12|}{4} = \frac{6}{4}$

$a_{24} = \frac{|3(2)-4(4)|}{4} = \frac{|6-16|}{4} = \frac{10}{4}$

$a_{31} = \frac{|3(3)-4(1)|}{4} = \frac{|9-4|}{4} = \frac{5}{4}$

$a_{32} = \frac{|3(3)-4(2)|}{4} = \frac{|9-8|}{4} = \frac{1}{4}$

$a_{33} = \frac{|3(3)-4(3)|}{4} = \frac{|9-12|}{4} = \frac{3}{4}$

$a_{34} = \frac{|3(3)-4(4)|}{4} = \frac{|9-16|}{4} = \frac{7}{4}$

$$\therefore B = \begin{bmatrix} 1/4 & 5/4 & 9/4 & 13/4 \\ 2/4 & 2/4 & 6/4 & 10/4 \\ 5/4 & 1/4 & 3/4 & 7/4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 2 & 6 & 10 \\ 5 & 1 & 3 & 7 \end{bmatrix}$$

7) If A = $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^4.

Answer : $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & a+a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^3 := A^2 \times A = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & a+2a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$

$A^4 = A^3 \times A = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & a+3a \\ 0+0 & 0+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

$\therefore A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

8) Express the following matrices as the sum of a symmetric matrix and a skew-symmetric matrix:

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Answer : $A=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}$

$\therefore A^T=\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

$A+A^T=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}+\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

$=\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$

$A-A^T=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}-\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

$=\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}$

$(1)\Rightarrow A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$

$A=\frac{1}{2}\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}$

9) Evaluate : $\begin{vmatrix}2&4\\-1&2\end{vmatrix}$

Answer : $\begin{vmatrix}2&4\\-1&2\end{vmatrix}$ = (2 × 2) - (- 1 × 4) = 4 + 4 = 8.

10) Evaluate : $\begin{vmatrix}cos\theta&sin\theta\\-sin\theta&cos\theta\end{vmatrix}$

Answer : $\begin{vmatrix}cos\theta&sin\theta\\-sin\theta&cos\theta\end{vmatrix}$ = (cosθcosθ) - (-sinθsinθ) = cos$^2\theta$ + sin$^2\theta$ = 1.

11) Find |A| if A = $\begin{bmatrix}0&sin\alpha&cos\alpha\\sin\alpha&0&sin\beta\\cos\alpha&-sin\beta&0\end{bmatrix}$.

Answer : $\begin{bmatrix}0&sin\alpha&cos\alpha\\sin\alpha&0&sin\beta\\cos\alpha&-sin\beta&0\end{bmatrix}$ = 0M_{11} - sinα M_{12} + cosα M_{13}

= 0 − sinα (0 − cosα sinβ) + cosα (−sinα sinβ − 0) = 0.

12) If a, b, c and x are positive real numbers, then show that $\begin{vmatrix}(a^x+a^{-x})^2&(a^x-a^{-x})^2&1\\(b^x+b^{-x})^2&(b^x-b^{-x})^2&1\\(c^x+c^{-x})^2&(c^x-c^{-x})^2&1\end{vmatrix}$ is zero.

Answer : Applying $C_1\rightarrow C_1 - C_2$, we get $\begin{vmatrix}4&(a^x-a^{-x})^2&1\\4&(b^x-b^{-x})^2&1\\4&(c^x-c^{-x})^2&1\end{vmatrix}$ = 0, Since C_1 and C_3 are proportional.

13) Evaluate $\begin{vmatrix}2014&2017&0\\2020&2023&1\\2023&2026&0\end{vmatrix}$

Answer : $\begin{vmatrix}2014&2017&0\\2020&2023&1\\2023&2026&0\end{vmatrix}$ = $\begin{vmatrix}2014&2017-2014&0\\2020&2023-2020&1\\2023&2026-2023&0\end{vmatrix}$ = $\begin{vmatrix}2014&3&0\\2020&3&1\\2023&3&0\end{vmatrix}$ = 3 $\begin{vmatrix}2014&1&0\\2020&1&1\\2023&1&0\end{vmatrix}$

= - 3(2014 - 2023) = - 3 (- 9) = 27.

14) Without expanding, evaluate the following determinants:

$\begin{vmatrix}x+y&y+z&z+x\\z&x&y\\1&1&1\end{vmatrix}$

Answer : $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2$ we get

$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Take X+y+z from R_1, We get

$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = (x+y+z)(0) = 0$

Since R_1 and R_3 are proportional.

15) If the area of the triangle with vertices (- 3, 0), (3, 0) and (0, k) is 9 square units, find the values of k.

Answer : Area of the triangle = absolute value of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$9 = |\frac{1}{2}| \begin{matrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{matrix} | = |\frac{1}{2}(-k)(-3-3)|$

$\Rightarrow 9 = 3|k|$ and hence, $k = \pm 3.$

16) If (k, 2), (2, 4) and (3, 2) are vertices of the triangle of area 4 square units then determine the value of k.

Answer : Given vertices are (k. 2). (2, 4) and (3, 2)

And also Given Area of a triangle = 4 sq,units

We know that, area of $\triangle$ABC =absolute value of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\Rightarrow$ 4 = absolute of $\frac{1}{2}\begin{vmatrix} k & 2 & 1 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$\Rightarrow$ 4 = absolute value of $\frac{1}{2}$[k(4 - 2)-2(2 - 3) + 1(4 - 12)]

[Expanded along R_1]

$\Rightarrow$ 4 = absolute value of $\frac{1}{2}$[2k + 2 - 8]

$\Rightarrow$ 4 =absolute value of $\frac{1}{2}$[2k - 6]

$\Rightarrow$ 4 = $\pm\frac{1}{2}$(2k - 6)

Case (i) when 4 =$\frac{1}{2}$(2k - 6)

$\Rightarrow$ 8 = 2k - 6

$\Rightarrow$14 = 2k

$\Rightarrow$ k =7

Case(ii) When 4 = -$\frac{1}{2}$(2k - 6)

$\Rightarrow$ 8 = -2k + 6

$\Rightarrow$ 8 - 6 = -2k

$\Rightarrow$ 2 = -2k

$\Rightarrow$ k = -1

$\therefore$ The values of k are -1 or 7.

17) Determine the values of a so that the following matrices are singular: A = $\begin{bmatrix} 7 & 3 \\ -2 & a \end{bmatrix}$

Answer : Given A is singular

$\therefore |A| = 0$

$\begin{vmatrix} 7 & 3 \\ -2 & a \end{vmatrix} = 0$

7a + 6 = 0

7a = -6

$a = \frac{-6}{7}.$

18) Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$

Answer : LHS = $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$

Multiply R_1, R_2, R_3 by a, b, c respectively, we get

$= \frac{1}{abc}\begin{vmatrix} ab+ac & abc & ab^2c^2 \\ bc+ab & abc & c^2a^2b \\ ac+bc & abc & a^2b^2c \end{vmatrix}$

Take abc from C_2 and C_3

$= \frac{(abc)(abc)}{abc}\begin{vmatrix} ab+ac & 1 & bc \\ bc+ab & 1 & ca \\ ac+bc & 1 & ab \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$

$= (abc)\begin{vmatrix} ab+ac+bc & 1 & bc \\ ab+bc+cb & 1 & ca \\ ab+bc+ca & 1 & ab \end{vmatrix}$

$= (abc)(ab+bc+ca)\begin{vmatrix} 1 & 1 & bc \\ 1 & 1 & ca \\ 1 & 1 & ab \end{vmatrix}$

$= abc(ab+bc+ca)(0)$

= 0 [ince C_1 and C_2 are proportional]

= RHS

Hence Proved.

19) Identify the singular and non-singular matrices:

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Answer : A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

|A| = $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

= 1 (45- 48) - 2 (36- 42) + 3 (32 - 35)

= 3 -2 (-6) + 3 (-3)

= -3 + 12 - 9 = -12 + 12 = 0

∴ A is a zero covariance matrix.

20) Identify the singular and non-singular matrices:

$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

Answer : A = $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

|A| = $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

= 2 (0 -20) + 3 (-42 - 4) + 5 (30 - 0)

= -40 + 3 x -46 + 150

= -40 - 138 + 150

= -178 + 150 = -28 $\neq$ 0

∴ A is a nonzero covariance matrix.

21) If A^T = $\begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$, verify $(A - B)^T = A^T - B^T$

Answer : Verify $(A - B)^T = A^T - B^T$

$$A - B = \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & -5 & 5 \end{bmatrix}$$

$$(A - B)^T = \begin{bmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{bmatrix} \text{ --(4)}$$

$$A^T - B^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{bmatrix} \text{ ----(5)}$$

From (4) and (5), $(A - B)^T = A^T - B^T$

22) Find x, y, z and w such that $\begin{bmatrix} x-y & 2z+w \\ 2x-y & 2x+w \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}$

Answer : Given $\begin{bmatrix} x-y & 2z+w \\ 2x-y & 2x+w \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}$

Equating the corresponding entries on both sides, we get

$x - y = 5$ ... (1)

$(-)\ (+)\qquad (-)$

$2x - y = 12$ ... (2)

$-x = -7 \quad \Rightarrow x = 7$

Substituting x=7 in (1) we get,

7-y = 5 $\Rightarrow$y=2

2z + w = 3

2x + w =15 ....(3)

2(7) + w =15 $\Rightarrow$4+w=15...(4)

$\Rightarrow$ w=1

Substituting w=1 in (3) we get,

2z + 1 =3 $\Rightarrow$2z= 2 $\Rightarrow$z = 1

$\therefore$x=7, y=2, z=1 and w=1

23) Solve $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

Answer : Given $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2 & 3x \\ y^2 & -6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

$\Rightarrow x^2-3x = -2$ and $y^2-6y=9$

$\Rightarrow x^2-3x+2 = 0$

$\Rightarrow$(x-1)(x-2) = 0

$\Rightarrow$x = 1,2

Also, $y^2-6y-9 = 0$

$y = \frac{6\pm\sqrt{36+36}}{2} = \frac{6+6\sqrt{2}}{2} = \frac{3(3\pm3\sqrt{2})}{2}$

$y=3 \pm 3\sqrt{2}$

x =1, 2 and $y= 3\pm3\sqrt{2}$

24) In a certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and 1 section of Express the given information as a column matrix. Using sclar multiplication find the total number of each kind in all the colleges.

Answer : Let A be the required column matrix

$$\text{Then } A = \begin{matrix} Peons \\ Clerks \\ Typist \\ Section\ officer \end{matrix} \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$

Since there are 30 collegs,

$$30\ A = 30 \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 450 \\ 180 \\ 30 \\ 30 \end{bmatrix}$$

25) Using properties of determinant, show that $\triangle = \begin{vmatrix} cosec^2\theta & -cot^2\theta & 1 \\ cot^2\theta & -cosec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$

Answer : Applying $C_1 \rightarrow C_1 - C_2$ we get

$$\triangle = \begin{vmatrix} cosec^2\theta & -cot^2\theta & 1 \\ cot^2\theta & -cosec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = \begin{vmatrix} 1 & cot^2\theta & 1 \\ -1 & cosec^2\theta & -1 \\ 2 & 40 & 2 \end{vmatrix}$$

= 0 $[\therefore C_1 \equiv C_2]$

26) Prove that $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)$

Answer : $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - R_3 \end{matrix}$

= $(a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$ Take (a - b) and (b - c) from R_1 and R_2 respectively.

= (a - b) (b - c) [(1) (b + c) - (1) (a + b)] = (a - b) (b - c) (c - a)

27) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = xy$

Answer : $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} \begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix}$

= xy [$\because$ upper diagonal matrix]

28) Identify the singular and non-singular matrix

(i) $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -2 & -4 & -6 \end{bmatrix}$

Answer : If the determinant value of a square matrix is zero it is called a singular matrix. Otherwise it is non-singular.

(i) Now $\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix} = 1\begin{vmatrix} 9 & 16 \\ 16 & 25 \end{vmatrix} - 4\begin{vmatrix} 4 & 16 \\ 9 & 25 \end{vmatrix} + 9\begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix}$

= 1 (225 - 256) - 4 (100 - 144) + 9 (64 - 81)

= 1 (-31) - 4 (-44) + 9 (-17)

= -31 + 176 -153 = -184 + 176 = -8 ≠ 0

$\therefore \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ is a non-singular matrix.

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -2 & -4 & -6 \end{bmatrix} = (-2)\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = (-2)(0) = 0 [\because R_1 = R_3]$

$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -2 & -4 & -6 \end{vmatrix}$ is a singular matrix.

29) If $\begin{bmatrix} x-1 & 2 & y-5 \\ z & 0 & 2 \\ 1 & -1 & 1+a \end{bmatrix} = \begin{bmatrix} 1-x & 2 & -y \\ 2 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ then find the values of x, y, z and a.

Answer : From the equality of matrices.

$x-1 = 1-x; y-5 = -y; z = 2; 1+a = 1$

Hence $x = 1; y = \frac{5}{2}; z = 2,\ a = 0$

30) If $A = \begin{bmatrix} 2 & 3 & 1 \\ 6 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ then find the matrix such that A + B - X = 0. What is the order of the matrix X

Answer : A and B are matrices of the same order 2×3 .

If A + B - X is to be defined, the order of X also must be 2×3

$$A + B - X = 0 \Rightarrow X = A + B$$

$$\therefore X = \begin{bmatrix} 2 & 3 & 1 \\ 6 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 0 \\ 6 & -2 & -8 \end{bmatrix}$$