QB365 Question Bank Software Study Materials

Sets, Relations and Functions Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Write the following in roster form.
The set of all positive roots of the equation (x-1)(x+1)(x^2-1) = 0.

Answer : The set of all positive roots of the equation (x-1)(x+1)(x^2-1)=0.
Let B = { the set of positive roots of the equation (x-1)(x+10(x^2-1)=0}
⇒ x = 1, -1
B = {1}.

2) Write the following in roster form {x∈N : 4x + 9 < 52}

Answer : {x ∈ N : 4x + 9 < 52}
Let C = {x∈N:4x + 9 < 52}
⇒ C = {x∈N:4x < 52 - 9}
⇒ C = {x∈N: 4x < 43}
⇒ C = $\left\{x \in N : x < \frac{43}{4}\right\}$ ⇒ C = {x ∈ N : x < 10.75}
⇒ C = {1,2,3,4,5,6,7,8,9,10}.

3) Write the following in roster form.
$\left\{x : \frac{x-4}{x+2} = 3, x \in R - \{-2\}\right\}$

Answer : $\left\{x : \frac{x-4}{x+2} = 3, x \in R - \{-2\}\right\}$
Let D = $\left\{x : \frac{x-4}{x+2} = 3, x \in R - \{-2\}\right\}$
⇒ D = {x: x - 4 = 3x + 6, x ∈ R}
⇒ D = {x:-4-6 = 3x-x, x ∈ R}
⇒ D = {x:2x = -10, x ∈ R}
⇒ D = {x:x = -5, x∈R}
⇒ D = {-5}

4) Write the set {-1, 1} in set builder form.

Answer : Let p = {-1, 1}
⇒ P = {x ∈ R : x is a root of the equation x^2-1 = 0}

5) State whether the following sets are finite or infinite.
{x ∈ N : x is an even prime number}

Answer : Let A = { x ∈ N: x is an even prime number}
⇒ A = {2}
⇒ A is a finite set.

6) State whether the following sets are finite or infinite.
{x ∈ N : x is an odd prime number}

Answer : Let B = {x∈N : x is an odd prime number}
⇒ B = {3, 5, 7, 11, ...}
⇒ B is an infinite set.

7) State whether the following sets are finite or infinite.
{x ∈ R : x is a rational number}

Answer : Let D = {x ∈ R : x is a rational number}

Answer : Let D = {x ∈ R : x is a rational number}

⇒ D = {set of all rational number}

⇒ D is an infinite set.

8) State whether the following sets are finite or infinite.

{x ∈ N:x is a rational number}

Answer : Let N = {x ∈ N: x is a rational number}

$\Rightarrow N = \{\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \ldots . \infty\}$

⇒ N ia an infinite set.

9) By taking suitable sets A, B, C, verify the following results:

A × (B∩C) = (A×B) ∩ (A×C)

Answer : A × (B∩C) = (A×B) ∩ (A×C)

Let A = {1,2,3}, B = (4,5,6,7} C = {4,3,5,9} and ∪ = {1,2,3,4,5,6,7,8,9}

LHS = A x (B∩C)

= A × {4,5}   [∴B∩C = {4, 5}]

= {1,2,3} × {4,5}

= {(1,4) (1,5) (2,4) (2,5) (3,4) (3,5)}......(1)

A x B = {1,2,3} x {4,5,6,7}

= {(1,4) (1,5) (1,6) (1,7) (2,4) (2,5) (2,6) (2,7) (3,4) (3,5) (3,6) (3,7)}

A × C = {1,2,3} × {3,4,5,6,9}

= {(1,3) (1,4) (1,5) (1,9) (2,3) (2,4) (2,5) (2,6) (2,9) (3,3) (3,4) (3,5) (3,9)}

RHS = (A × B) ∩ (A × C) = {(1,4) (1,5) (2,4) (2,5) (3,4) (3,5) }

From (1) and (2), LHS = RHS. Hence Verified....(2)

10) By taking suitable sets A, B, C, verify the following results:

A × (B∪C) = (A×B) ∪ (A×C)

Answer : (B∪C) = {3, 4, 5, 6 ,7, 9}

Now, A×(B∪C) = {1, 2, 3} ×{3, 4, 5, 6, 7, 9}

= {(1,3)(1,4)(1,5)(1,6)(1,7)(1,9)(2,3)(2,4)(2,5)(2,6)(2,7)(2,9)(3,3)(3,4)(3,5)(3,6)(3,7)(3,9)} .....(1)

Now A×B = {1,2,3} × {4,5,6,7}

= {(1,4)(1,5)(1,6)(1,7)(2,4)(2,5)(2,6)(2,7)(3,4)(3,5)(3,6)(3,7)}

A×C = {1,2,3} × {3,4,5,9}

= {(1,3)(1,4)(1,5)(1,9)(2,3)(2,4)(2,5)(2,9)(3,3)(3,4)(3,5)(3,9)}

RHS(A×B)∪(A×C) = {(1,3)(1,4)(1,5)(1,6)(1,7)(1,9)(2,3)(2,4)(2,5)(2,6)(2,7)(2,9)(3,3)(3,4)(3,5)(3,6)(3,7)(3,9)} .....(2)

From (1) & (2), LHS = RHS

Hence verified

11) By taking suitable sets A, B, C, verify the following results:

(A× B)∩(B×A) = (A∩B) × (B∩A)

Answer : (A× B) = {(1,4) (1,5) (1,6) (1,7) (2,4) (2,5) (2,6) (2,7) (3,4) (3,5) (3,6) (3,7)}

(B×A) = {(4,1) (4,2) (4,3) (5,1) (5,2) (5,3) (6,1) (6,2) (6,3) (7,1) (7,2) (7,3)}

LHS = (A×B)∩(B×A) = { }....(1)

(A∩B) = { }, (B∩A) = { }

∴ RHS = (A∩B) × (B∩A) = { }.....(2)

From (1) and (2), LHS = RHS

12) By taking suitable sets A, B, C, verify the following results:

C-(B-A) = (C∩ A) ∪ (C∩B')

Answer : B-A = {4, 5, 6, 7}

LHS = C-(B-A) = {3, 9}...(1)

C∩A = {3}

B' = {1,2,3,8,9}

C∩B' = {3, 9}

RHS = (C∩A)∪ (C∩B') = {3, 9}...(2)

From (1) and (2), LHS = RHS

13) Justify the trueness of the statement "An element of a set can never be a subset of itself".

Answer : Let P = {a, b, c, d}

Each and every element of the set P can be a subset of the set itself

Eg: {a}, {b}, {c}, {d}.

Hence, the given statement is not true.

14) Discuss the following relations for reflexivity, symmetricity and transitivity :

Let A be the set consisting of all the female members of a family. The relation R defined by "aRb if a is not a sister of b".

Answer : Given relation is aRb if a is not a sister of b.

Let a, b, C $\in$ A.

Reflexivity : aRa $\Rightarrow$ a is not a sister of a

$\therefore$ R is reflexive.

Symmetricity: aRb $\Rightarrow$ bRa

a is not a sister of b $\Rightarrow$ b is not a sister of a.

$\therefore$ R is symmetric.

Transitivity : aRb and bRC $\Rightarrow$ aRC

a is not a sister of b, b is not a sister of C [Eg : Mother is not a sister of daughter, daughter is not a sister of chithi, but mother is a sister of chithi.]

$\Rightarrow$ a is a sister of C.

$\therefore$ R is not transitive.

$\therefore$ R is reflexive, symmetric and but not transitive.

15) Discuss the following relations for reflexivity, symmetricity and transitivity :

On the set of natural numbers, the relation R is defined by "xRy if x + 2y = 1".

Answer : The relation R is defined by xRy if x + 2y = 1 for x, y $\in$ N.

Reflexivity : Let x, y $\in$ N

xRx $\Rightarrow$ x + 2x = 1 $\Rightarrow$ 3x = 1 $\Rightarrow$ x = $\frac{1}{3} \notin N$

$\therefore$ R is reflexive.

Symmetricity: xRy $\Rightarrow$ yRx for x, y $\in$ N

xRy $\Rightarrow$ x + 2y = 1 which is not possible for any values of x, Y $\in$ N

$\therefore$ R is not symmetric

Transitivity: xRy and yRz $\Rightarrow$ xRz.

xRy and yRz are not possible for any values of x, y, z $\in$ N

$\therefore$ R is not transitive.

$\therefore$ R is neither reflexive, nor symmetric and not transitive.

16) Prove that the relation "friendship" is not an equivalence relation on the set of all people in Chennai.

Answer : Let a, b, c are people in Chennai

Reflexivity: "a" is a friend of "a" $\Rightarrow$ a R a $\Rightarrow$ R is not reflexive.

Symmetric: a is friend of b $\Rightarrow$ b is the friend of a.

$\therefore$ aRb $\Rightarrow$ bRa $\Rightarrow$ R is symmetric

Transitive: a is the friend of b and b is the friend of c $\Rightarrow$ a need not be the friend of c.

$\therefore$ aRb $\Rightarrow$ bRc $\neq$ aRc $\Rightarrow$ R is not transitive

Hence, the relation "friendship" is not equivalent.

17) If X = {1, 2, 3, .. 10} and A = {1, 2, 3, 4, 5}, find the number of sets $B \subseteq X$ such that A - B = {4}.

Answer : For every subset C of {6, 7, 8, 9, 10}, let B = C $\cup$ {1, 2, 3, 5}. Then A - B = {4}. In other words, for every subset C of {6, 7, 8, 9, 10}, we have a unique set B so that A - B = {4}.

So number of sets $B \subseteq X$ such that A - B = {4} and the number of subsets of {6, 7, 8, 9, 10} are the same.

So the number of sets $B \subseteq X$ such that A - B = {4} is 2^5 = 32.

18) Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k.

Answer : Let A and B be the two sets with n(A) = m and n(B) = k. Since A contains more elements than B, we have m > k. From the given conditions we see that 2^m - 2^k = 112. Thus we get, 2k (2^{m-k} - 1) = $2^4 \times 7$.
Then the only possibility is k = 4 and 2^{m-k} - 1 = 7. So m - k = 3 and hence m = 7.

19) If $f : R - \{-1, 1\} \rightarrow R$ is defined by $f(x) = \frac{x}{x^2-1}$, verify whether f is one-to-one or not.

Answer : We start with the assumption f(x) = f(y). Then
$\frac{x}{x^2-1} = \frac{y}{y^2-1}$
$\Rightarrow x(y^2-1) = (x^2-1)$
$\Rightarrow xy^2 - x - yx^2 + y - 0 \Rightarrow (y-x)(xy+1) = 0$
This implies that x =y or xy = - 1. So if we select two numbers x and y so that xy = -1, then f(x) = f(y).
$f(x) = f(y) \cdot \left(2, -\frac{1}{2}\right), \left(7, -\frac{1}{7}\right), \left(-2, \frac{1}{2}\right)$ are some among the infinitely many possible pairs. Thus $f(2) = f\left(\frac{-1}{2}\right) = \frac{2}{3}$. That is, f(x) = f(y) does not imply x = y. Hence it is not one-to-one.

20) If $f : R \rightarrow R$ is defined as f(x) = $2x^2$-1, find the pre-image of 17, 4 and -2.

Answer : To find the pre-image of 17, we solve the equation $2x^2$ - 1 = 17. The two solutions of this equation, 3 and - 3 are the pre-images of 17 under f. The equation $2x^2$ - 1 = 4 yields $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$ as the two pre-images of 4. To find the pre-image of -2, we solve the equation $2x^2$-1 = -2. This shows that = $x^2 = -\frac{1}{2}$ which has no solution R because square of a number cannot be negative and hence - 2 has no pre-image under f.

21) If A = {x : x is a multiple of 5, x ≤ 30 and x ∈ N}
B = {1, 3, 7, 10, 12, 15, 18, 25} then find A∩B

Answer : Given A = {5, 10, 15, 20, 25, 30}
B = {1, 3, 7, 10, 12, 15, 18, 25}
Now A∩B = {10,15, 25}

22) If the function f : R→R be given by f(x) = x^2 + 2 and g(x) = 2x, find f o g and g o f

Answer : Given f(x) = x^2+2, g(x) = 2x
fog(x) = f(g(x)) = f(2x) = $(2x)^2$+2 = $4x^2$+ 2
gof(x) = g(f(x)) = g(x^2+2) = 2(x^2+2) = $2x^2$+ 4

23) On a set of natural numbers let R be the relation defined by aRb if a + 2b = 15. Write down the relation by listing all the pairs. Check whether it is reflexive, symmetric, transitive, equivalence.

Answer : (1,7) (3, 6) (5, 5) (7,4) (9,3) (11, 2) (13, 1) not reflexive, not symmetric, not transitive, not equivalence.

24) Let C be the set of all circles in a plane and define a circle C is related to a circle C', if the radius of C is equal to the radius of C'

Answer : In this example, it is easy to see that whenever a circle C_1 is related to C_2 then C_2 is also related to C_1. Hence the relation is symmetric, whereas in example 1 it does not hold as whenever 2 divides 4 then it does not imply 4 divides 2 as 4 does not divide 2.

25) Let A be the set consisting of children and elders of a family. Let R be the relation defined by aRb if a is a sister of b.

Answer : This relation is to be looked into carefully. A woman is not a sister of herself. So it is not reflexive. It is not symmetric also. Clearly it is not transitive. So it is not an equivalence relation. (If we consider the same relation on a set consisting only of females, then it becomes symmetric, even in this case it is not transitive).

26) Write the following sets in roster form, {x ∈ N; x^3< 1000}

Answer : A = {1, 2, 3, 4, 5,6, 7, 8, 9}

27) Write the following sets in roster form, {The set of positive roots of the equation (x^2- 4)(x^3- 27) = 0}.

Answer : B = {2, 3}

28) If A = { 0, 1, 2, 3, 4, 5, 6, 7 } is a set. Then,

Answer : A = { x | x is a whole number less than or equal to 7 } is the set-bilder form of A.

29) If Ax A has 9 elements, S = {(a, b) ∈ A × A: a > b}; (2, -1) and (2, 1) are two elements, then find the remaining elements of S.

Answer : $n(A \times A) = 9$

$\Rightarrow n(A) = 3$

$S = \{(a, b) \in A \times A : a > b\}$

$A = \{-1, 1, 2\}$

$A \times A = \{(-1, -1), (-1, 1), (-1, 2), (1, -1), (1, 1), (1, 2), (2, 1), (2, 1) (2, 2)\}$

$\therefore S = \{(1, -1), (2, -1), (2, 1)$

30) Show that the relation B on the set A = {1, 2, 3} given by {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

Answer : Since $1, 2, 3 \in A$ and $(1,1), (2, 2), (3, 3) \in R$

i.e, $\forall\ a \in A, (a, a) \in R$

$\therefore$ R is reflexive

Observe that $(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric

Also, $(1, 2) \in R$ and (2, 3) e Rbut $(1, 3) \notin R$

$\therefore$ R is not transitive.