

QB365 Question Bank Software Study Materials

Trigonometry Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 50

2 Marks

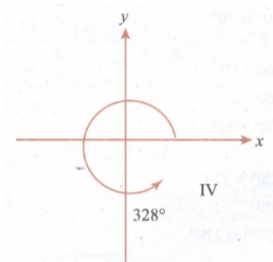
30 x 2 = 60

- 1) Identify the quadrant in which an angle of each given measure lies; 328°

Answer : 328°

$$328^\circ = 270^\circ + 58^\circ$$

$\therefore 328^\circ$ lies in the IV quadrant

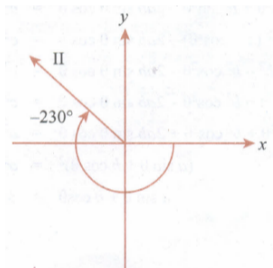


- 2) Identify the quadrant in which an angle of each given measure lies; -230°

Answer : -230°

$$-230^\circ = -180^\circ - 50^\circ$$

$\therefore -230^\circ$ lies in the II quadrant



- 3) Find the value of $\cos 105^\circ$

Answer : $\cos 105^\circ = \cos (60^\circ + 45^\circ)$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

- 4) Prove that $\cos (30^\circ + x) = \frac{\sqrt{3} \cos x - \sin x}{2}$

Answer : $\cos (30^\circ + x) = \frac{\sqrt{3} \cos x - \sin x}{2}$

$$\cos (30^\circ + x) = \cos 30^\circ \cos x - \sin 30^\circ \sin x$$

$$= \frac{\sqrt{3}}{2} \cdot \cos x - \frac{1}{2} \sin x = \frac{\sqrt{3} \cos x - \sin x}{2}$$

- 5) Prove that $\sin (\pi + \theta) = -\sin \theta$

Answer : $\sin (\pi + \theta) = -\sin \theta$

$$\sin (\pi + \theta) = \sin \pi \cos \theta + \cos \pi \sin \theta$$

$$= (0) \cos \theta + (-1) \sin \theta$$

$$= 0 - \sin \theta$$

$$\sin (\pi + \theta) = -\sin \theta$$

- 6) Find the principal value of $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$.

Answer : Let $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = y$, where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\Rightarrow \sin y = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin y = \sin \frac{\pi}{4}$$

$$\Rightarrow y = \frac{\pi}{4}$$

Thus the principal value of $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = \frac{\pi}{4}$.

7) Find the principal value of $\operatorname{cosec}^{-1}(-1)$

Answer : Let $\operatorname{cosec}^{-1}(-1) = y$, where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\Rightarrow -1 = \operatorname{cosec} y$$

$$\Rightarrow \operatorname{cosec} y = -\operatorname{cosec} \left(\frac{\pi}{2}\right)$$

$$\Rightarrow \operatorname{cosec} y = \operatorname{cosec} -\left(\frac{\pi}{2}\right) \quad [\because \operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta]$$

$$\Rightarrow y = -\left(\frac{\pi}{2}\right)$$

Thus, the principal value of $\operatorname{cosec}^{-1}$ is $-\left(\frac{\pi}{2}\right)$.

8) Express each of the following angles in radian measure

$$150^\circ$$

Answer : 150°

$$150^\circ = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$$

9) Express each of the following angles in radian measure

$$330^\circ$$

Answer : 330°

$$330^\circ = 330 \times \frac{\pi}{180} = \frac{11\pi}{6}$$

10) Find the degree measure corresponding to the following radian measure; $\frac{\pi}{9}$

Answer : $\frac{\pi}{9}$

$$\frac{\pi}{9} = \frac{\pi}{9} \times \frac{180}{\pi} = 20^\circ$$

11) Express each of the following as a sum or difference. $\sin 35^\circ \cos 28^\circ$

Answer : $\sin 35^\circ \cos 28^\circ = \frac{1}{2} [2 \sin 35^\circ \cos 28^\circ]$

$$= \frac{1}{2} [\sin (35 + 28) + \sin (35 - 28)]$$

$$= \frac{1}{2} [\sin 63^\circ + \sin 7^\circ]$$

12) Express each of the following as a sum or difference. $\sin 4x \cos 2x$

Answer : $\sin 4x \cos 2x = \frac{1}{2} [\sin (4x + 2x) + \sin (4x - 2x)]$

$$= \frac{1}{2} [\sin 6x + \sin 2x]$$

13) Express each of the following as a sum or difference. $\cos 5\theta \cos 2\theta$

Answer : $\cos 5\theta \cos 2\theta = \frac{1}{2} [\cos (5\theta + 2\theta) + \cos (5\theta - 2\theta)]$

$$= \frac{1}{2} [\cos 7\theta + \cos 3\theta]$$

14) A football player can kick a football from ground level with an initial velocity of 80 ft/second. Find the maximum horizontal distance the football travels and at what angle? (Take $g = 32$)

Answer : The formula for horizontal distance R is given by

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(80 \times 80) \sin 2\alpha}{32}$$

$$= 10 \times 20 \sin 2\alpha$$

Thus, the maximum distance is 200ft.

Hence, he has to kick the football at an angle of $\alpha = 45^\circ$ to reach the maximum distance.

15) Convert : 18° to radians.

Answer : Now, $180^\circ = \pi$ radians gives $1^\circ = \frac{\pi}{180}$ radians

$$18^\circ = \frac{\pi}{180} \times 18 \text{ radians} = \frac{\pi}{10} \text{ radians.}$$

16) Convert : 6 radians to degrees.

Answer : We know that π radians = 180° and thus,

$$6 \text{ radians} = \left(\frac{180}{\pi} \times 6\right)^\circ \approx \left(\frac{7 \times 180}{22} \times 6\right)^\circ = \left(343 \frac{7}{11}\right)^\circ$$

17) Find the general solution of $\sin \theta = -\frac{\sqrt{3}}{2}$.

Answer : The general solution of $\sin \theta = \sin \alpha$, $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, is $\theta = n\pi + (-1)^n \alpha$, $n \in Z$

$$\sin \theta = -\frac{\sqrt{3}}{2} = \sin(-\frac{\pi}{3}),$$

Thus the general solution is

$$\theta = n\pi + (-1)^n (-\frac{\pi}{3}) = n\pi + (-1)^{n+1} \frac{\pi}{3}; n \in Z \dots (i)$$

18) Simplify $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$.

Answer : We have $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{2\cos(\frac{75^\circ+15^\circ}{2})\sin\frac{75^\circ-15^\circ}{2}}{2\cos(\frac{75^\circ+15^\circ}{2})\cos(\frac{75^\circ-15^\circ}{2})}$
 $= \frac{2\cos 45^\circ \sin 30^\circ}{2\cos 45^\circ \cos 30^\circ} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

19) Find the principal value of $\sin^{-1}(\frac{\sqrt{3}}{2})$

Answer : Let $\sin^{-1}(\frac{\sqrt{3}}{2}) = y$, where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\Rightarrow \sin y = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow y = \frac{\pi}{3}$$

Thus, the principal value of $\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$

20) Find the principal value of $\tan^{-1}(\frac{-1}{\sqrt{3}})$

Answer : Let $\tan^{-1}(\frac{-1}{\sqrt{3}}) = y$, where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\tan y = -\frac{1}{\sqrt{3}} \Rightarrow \tan y = \tan(-\frac{\pi}{6}) \Rightarrow y = -\frac{\pi}{6}$$

Thus the principal value of $\tan^{-1}(\frac{-1}{\sqrt{3}}) = -\frac{\pi}{6}$

21) Find the radian measures $40^\circ 20'$

Answer : Clearly $20' = (\frac{20}{60})^\circ = \frac{1}{3}^\circ$

$$\therefore 40^\circ 20' = (40\frac{1}{3})^\circ = (\frac{121}{3})^\circ = (\frac{121}{3} \times \frac{\pi}{180})^c = (\frac{121\pi}{540})^c$$

22) Find the value of $\tan \frac{\pi}{12}$.

Answer : $\tan(\frac{\pi}{12}) = \tan(\frac{\pi}{4} - \frac{\pi}{6})$

$$= \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}} \quad \left[\because \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1(\frac{1}{\sqrt{3}})} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \quad \left[\because \tan \frac{\pi}{4} = 1, \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \right]$$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{(\sqrt{3})^2 - 2(1)(\sqrt{3}) + 1}{(\sqrt{3})^2 - 1^2} = \frac{3 - 2\sqrt{3} + 1}{2} \quad \left[\because \text{conjugating the denominator} \right]$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{4 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2} = 2 - \sqrt{3}$$

23) Prove that $2\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

Answer : LHS = $\cos\left(\frac{\pi}{13}\right)\cos\left(\frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$$\cos\left(\frac{9\pi}{13} + \frac{\pi}{13}\right) + \cos\left(\frac{9\pi}{13} - \frac{\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right) \quad [2\cos A \cos B = \cos(A+B) + \cos(A-B)]$$

$$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right) = -\cos\left(\frac{3\pi}{13}\right) - \cos\left(\frac{5\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$$

$$= 0 = \text{RHS}$$

$[\because \cos(\pi - A) = -\cos A]$

24) Find the area of a triangle ABC in which $\angle A = 60^\circ$, $b = 4$ cm and $c = \sqrt{3}$ cm.

Answer : Area of triangle ABC is given by

$$\Delta = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}(4)(\sqrt{3})\sin 60^\circ = \left(\frac{1}{2}\right)(4)(\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)$$

$$\Delta = 3 \text{ sq.cm}$$

25) Find $\sin 15^\circ$, $\cos 15^\circ$ and $\tan 15^\circ$. Hence evaluate $\cot 75^\circ + \tan 75^\circ$.

Answer : $\frac{\sqrt{3}-1}{2\sqrt{2}}$, $\frac{\sqrt{3}+1}{2\sqrt{2}}$, $\frac{\sqrt{3}-1}{\sqrt{3}+1}$, 4

26) Evaluate: $\sin^{-1}\left(\cos^{-1}\frac{3}{5}\right)$

Answer : $\frac{4}{5}$

27) Evaluate: $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer : $\frac{3\pi}{4}$

28) Find the principal solutions of the equation $\sin x = \frac{\sqrt{3}}{2}$

Answer : We know that,

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \text{ and}$$

$$\sin \frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$$

$$= \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

Therefore, principal solutions are $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

29) Express $\sin 5\theta \sin 4\theta$ as a sum or difference.

Answer : $\sin 5\theta \sin 4\theta = \frac{1}{2} [\cos (5\theta - 4\theta) - \cos (5\theta + 4\theta)]$
 $= \frac{1}{2} [\cos \theta - \cos 9\theta]$

30) If $A+B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$.

Answer : $A + B = 45^\circ$

$$\tan(A + B) = \tan 45^\circ$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$\text{L.H.S.} = (1 + \tan A)(1 + \tan B)$$

$$= 1 + \tan A + \tan B + \tan A \tan B$$

$$= 1 + 1 - \tan A \tan B + \tan A \tan B$$

$$= 2 = \text{R. H. S.}$$