QB365 Question Bank Software Study Materials

Two Dimensional Analytical Geometry Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Find the locus of P, if for all values of α the co-ordinates of a moving point P is (9 cos α 9 sin α)

Answer : (9 cos α, 9 sin α)

Let P (h, k) be any point on the required path

From the given information, we have

h = 9 cos α and k = 9 sin α

$\Rightarrow$ $\frac{h}{9}$ = cos α and $\frac{k}{9}$ = sin α

$\left(\frac{h}{9}\right)^2+\left(\frac{k}{9}\right)^2=cos^2\alpha+sin^2\alpha$

$\Rightarrow \frac{h^2}{81}+\frac{k^2}{81}=1 \quad [\because sin^2\alpha+cos^2\alpha=1]$

$\Rightarrow$ $h^2 + k^2 = 81$

$\therefore$ Locus of (h, k) is $x^2 + y^2 = 81$

2) Find the equation of the lines passing through the point (1, 1)

(i) with y-intercept (-4)

(ii) with slope 3

(iii) and (-2, 3)

(iv) and the perpendicular from the origin makes an angle 60° with x- axis.

Answer : (i) with y-intercept (-4)

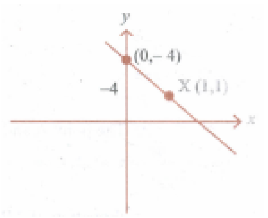
Equation of the line passing through (x, y) with y - intercept c is y = mx + c

Since the y - intercept is c = - 4, (0, -4) is also a point on the line.

Slope of line joining (1, 1) and (0, 4) is

$m=\frac{y_2-y_1}{x_2-x_3}=\frac{-4-1}{0-1}=\frac{-5}{-1}=5$

$\therefore$ Required equation is y = 5x - 4. [m = 5, c = -4]



(ii) with slope 3

Equation of the line passing through (1, 1) with slope 3 is

y -1 = 3 (x - x_1) [$\because$ y - y_1 = m(x - x_1)]

$\Rightarrow$ y - 1 = 3x - 3

$\Rightarrow$ 3x - y = -1 + 3

$\Rightarrow$ 3x - y = 2

(iii) and (-2, 3)

Equation of the line passing through (1, 1) and (-2, 3)

$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$\Rightarrow \frac{y-1}{3-1}=\frac{x-1}{-2-1}$

$\Rightarrow \frac{y-1}{2}=\frac{x-1}{-3}$

$\Rightarrow$ -3y + 3 = 2x - 2

$\Rightarrow$ 2x + 3y = 3 + 2

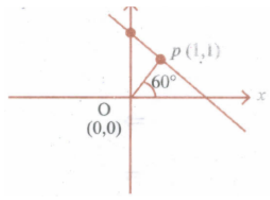
$\Rightarrow$ 2x + 3y = 5

(iv) and the perpendicular from the origin makes an angle 60° with x- axis .

Given α = 60°

Perpendicular distance p = distance between op

= $\sqrt{(1-0)^2+(1-0)^2}=\sqrt{2}$



∴ Required equation in normal form is x cos α + y sin α = p

⇒ x cos 60° +y sin 60° = √2

⇒ $x(\frac{1}{2}) + y\frac{\sqrt{3}}{2} = \sqrt{2}$

⇒ $\frac{x+\sqrt{3}y}{2} = \sqrt{2}$

⇒ x + √3y = 2√2

3) Show that the lines are 3x + 2y + 9 = 0 and 12x + 8y - 15 = 0 are paralle llines.

Answer : If the equation of two lines are in general form as a_1 x + b_1 y_1 + c = 0 and a_2x + b_2y + c_2 = 0

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ $or\ a_1b_2 = a_2b_1$

Given lines are 3x + 2y + 9 = 0 and 12x + 8y - 15 = 0

$\frac{3}{12} = \frac{2}{8}$

$\Rightarrow \frac{1}{4} = \frac{1}{4}$

Hence the given lines are parallel.

4) Find the equation of the straight line parallel to 5x - 4y + 3 = 0 and having x-intercept 3.

Answer : Since x-intercept is 3, A (3, 0) will be a point on the required line.

Any line parallel to 5x - 4y + 3 = 0 will be .of the form 5x - 4y + k = 0

Substituting the point (3, 0) we get

+15 - 0 + k = 0

⇒ k = -15

∴ Required equation of the line is 5x - 4y + -15 = 0

5) Find the distance between the line 4x + 3y + 4 = 0 and a point (-2, 4)

Answer : (-2, 4)

Distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\pm\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$

Distance from the point (-2, 4) to the line 4x + 3y + 4 = 0 is

$\pm\left|\frac{4(-2)+3(4)+4}{\sqrt{4^2+3^2}}\right|$

$= \pm\frac{(-8+12+4)}{\sqrt{25}} = \frac{8}{5}$ Unit

6) Write the equation of the lines through the point (1,-1)

(i) parallel to x + 3y - 4 = 0

(ii) perpendicular to 3x + 4y = 6

Answer : (i) Any line parallel to x + 3y - 4 = 0 will be of the form x + 3y + k = 0.

This line passes through (1, -1)

∴ 1 + 3 (-1) + k = 0

⇒ 1-3 + k = 0

⇒ k - 2 = 0

⇒ k = 2

∴ The required line is x + 3y + 2 = 0

(ii) Any line perpendicular to 3x + 4y - 6 = 0 will be of the form 4x - 3y + k = 0.

This line passes through (1, -1)

∴ 4(1) - 3(1) + k = 0

⇒ 4 + 3 + k = 0

⇒ k = -7

∴ The required line is 4x - 3y - 7 = 0.

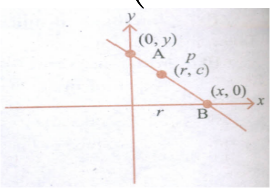
7) If p (r, c) is mid - point of a line segment between the axes, then show that $\frac{x}{r} + \frac{y}{c} = 2$

Answer : Since A and B are the points on the axes, its co-ordinate are A(x, 0) and B(0, y)

Given that p(r, c) is the mid-point of Ab.

∴ Using mid-point formula,

$(r, c) = \left(\frac{x+0}{2}, \frac{0+y}{2}\right)$



$\Rightarrow \quad r = \frac{x}{2} and \quad c = \frac{y}{2}$

$\Rightarrow \quad x = 2r \ and \ y = 2x$

$\therefore \ The \ point \ A \ and \ B \ are \begin{pmatrix} x_2 & y_2 \\ 0 & 2c \end{pmatrix} and \begin{pmatrix} x_1 & y_1 \\ 2r & 0 \end{pmatrix}$

$\therefore \quad Equation \ of \ AB \ is \frac{y-0}{2c-0} = \frac{x-2r}{0-2r}$

$\Rightarrow \quad \frac{y}{2c} = \frac{x-2r}{-2r} \quad \Rightarrow \quad \frac{y}{c} = \frac{x-2r}{-r}$

$\Rightarrow -ry = cx - 2rc \ \Rightarrow \ cr + xy = -2rc$

$\Rightarrow \ cx + ry = 2rc$

Dividing by rc, we get, $\frac{cx}{rc} + \frac{ry}{rc} = \frac{2rc}{rc} \Rightarrow \frac{x}{r} + \frac{y}{c} = 2$ Hence proved.

8) Find the distance between the parallel lines

$12x + 5y = 7$ and $12x + 5y + 7 = 0$.

Answer : $12x + 5y = 7$ and $12x + 5y + 7 = 0$

Given parallel lines are $12x + 5y = 7$ and $12x + 5y + 7 = 0$

Here a = 12, b = 5, $c_1 = -7$ and $c_2 = 7$

Distance between parallel lines = $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

= $\left|\frac{-7-7}{\sqrt{12^2+5^2}}\right| = \left|\frac{-14}{\sqrt{169}}\right|$

Distance between parallel lines = $\frac{14}{13}$

9) Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

Answer : Given equation of pair of lines is $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$

Here a = 4, b = 1, 2h = 4, 2g = -6, 2f = -3 and c = -4

h = 2, g = -3 , $f = \frac{-3}{2}$

The condition to represent pair of parallel lines is

$h^2 - ab = 0$

$\Rightarrow$ $2^2 - 4(1) = 0$

$\Rightarrow$ 4 - 4 = 0

$\Rightarrow$ 0 = 0

Also, condition to represent pair of lines is

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 4 & 2 & -3 \\ 2 & 1 & -3 \\ -3 & \frac{-3}{2} & -4 \end{vmatrix} = 0$$

R_1 we get

$\Rightarrow \left(-4\frac{-9}{4}\right) - 2\left(-8 - \frac{9}{2}\right) - 3(-3+3) = 0$

$\Rightarrow 4\left(\frac{-16-9}{4}\right) - 2\left(\frac{-16-9}{2}\right) = 0$

$\Rightarrow$ -25 + 25 = 0

$\Rightarrow$ 0 = 0

Hence the given equation represents a parallel lines.

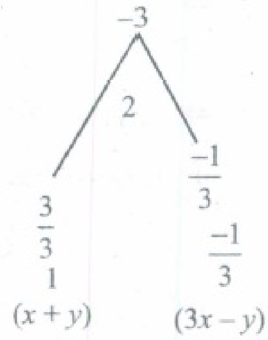
10) Find the separate equation of the following pair of straight lines.

$3x^2 + 2xy - y^2 = 0$

Answer : $3x^2 + 2xy - y^2 = 0$

Consider $3x^2 + 2xy - y^2 = 0$

$\Rightarrow$ (x +y) (3x - y) = 0 [By factorizing]



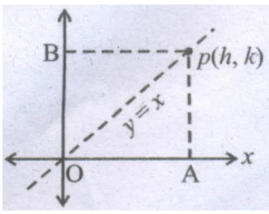
Hence the separate equations are

x + y = 0 and 3x - y = 0

11) Find the locus of a point which moves such that its distance from the x-axis is equal to the distance from the y-axis.

Answer : Let P (h, k) be a point on the locus.

Let A and B be the foot of the perpendiculars drawn from the point P on the x-axis and the y-axis respectively.



Therefore P is (OA, OB) = (BP, AP) = (h, k)

Given that AP = BP

$\Rightarrow$ k = h

replacing hand k by substituting h = x and k = y

The locus of P is y = x, is a line passing through the origin.

12) Find the path traced out by the point $\left(ct, \frac{c}{t}\right)$, here t ≠ 0 is the parameter and c is a constant.

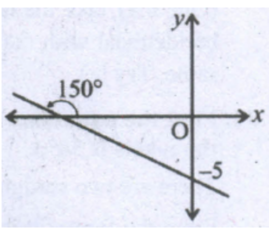
Answer : Let P (h, k) be a point on the locus, From the given information, we have h = ct and k = $\frac{c}{t}$. To eliminate t, taking product of these two equations

(h)(k) = (ct)($\frac{c}{t}$) $\Rightarrow$ hk = c^2

Therefore, the required locus is xy = c^2

13) Find the equation of a straight line cutting an intercept of 5 from the negative direction of the y-axis and is inclined at an angle 150^0 to the x-axis.

Answer : Given that the negative y intercept is 5 i.e., b = -5 and a = 150^0,



Slope m = tan 150^0 = tan(180° - 30^0) = -tan 30^0 = $-\frac{1}{\sqrt{3}}$

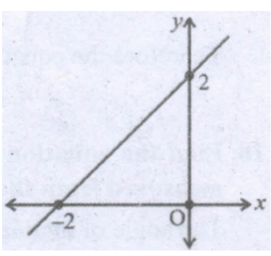
Slope and intercept form of the equation is y = mx + b

That is y = $-\frac{1}{\sqrt{3}}$x - 5

$$x + \sqrt{3}y + 5\sqrt{3} = 0$$

14) Find the equation of the straight line passing through (-1, 1) and cutting off equal intercepts, but opposite in signs with the two coordinate axes.

Answer : Let the intercepts cut off from the axes be of lengths a and -a.



Equation of the line is of the form $\frac{x}{a} - \frac{y}{a} = 1$ $\Rightarrow$ x - y = a

Since it passes through (-1, 1)

(1) $\Rightarrow$ (-1) - (1) = a $\Rightarrow$ a = -2

Equation of the line is x - y + 2 = 0

15) Find the combined equation of the straight lines whose separate equations are x - 2y - 3 = 0 and x + y + 5 = 0.

Answer : The combined equation of straight lines

(x - 2y - 3) (x + y + 5) = 0

x^2 + xy + 5x - 2xy - $2y^2$- 10y - 3x - 3y - 15 = 0

x^2- $2y^2$- xy + 2x -13y - 15 = 0

16) Find the locus of P, if for all values of α the co-ordinates of a moving point P is (9 cos α, 6 sin α)

Answer : (9 cos α , 6 sin α)

Let P (h, k) be any point on the required path

From the given information, we have

h 9 cos α and k = 6 sin α

$\Rightarrow$ $\frac{h}{9}$ = cos α and $\frac{k}{6}$ = sin α

To eliminate the parameter α

Squaring and adding we get

$\left(\frac{h}{9}\right)^2 + \left(\frac{k}{6}\right)^2 = cos^2\alpha + sin^2\alpha$

$\Rightarrow$ $\frac{h^2}{81} + \frac{k^2}{36} = 1$ [$\because$ $\cos^2 \alpha$ + $\sin^2 \alpha$ = 1]

$\therefore$ Locus of (h, k) is $\frac{x^2}{81} + \frac{y^2}{36} = 1$

17) Find the distance between the line 4x + 3y + 4 = 0 and a point.(7,−3)

Answer : (7,-3)

Distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\pm\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$

Distance from the point (7, -3) to the line 4x + 3y + 4 = 0. is

$= \pm\frac{4(7)+3(-3)+4}{\sqrt{4^2+3^2}}$

$= \pm\frac{(28-9+4)}{\sqrt{25}}$

$= \pm\left(\frac{23}{5}\right)$

$= \frac{23}{5}$units

18) Find the distance between the parallel lines.

3x - 4y + 5 = 0 and 6x - 8y - 15 = 0.

Answer : 3x - 4y + 5 = 0 and 6x - 8y - 15 = 0.

Given parallel lines are 3x - 4y + 5 = 0

$\Rightarrow$ 6x - 8y + 10 = 0 and 6x-8y-15 = 0 [Multiplied by 2]

Here a = 6, b = -8, c_1 = 10 and c_2 = -15

Distance between parallel lines = $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$ = $\left|\frac{10-(-15)}{\sqrt{6^2+(-8)^2}}\right|$

= $\left|\frac{25}{\sqrt{36+64}}\right| = \left|\frac{25}{10}\right| = \frac{5}{2}$

19) The sum of the squares of the distances of a moving point from two fixed points (a, 0) and (-0, 0) is equal to $2c^2$. Find the equation to its locus.

Answer : Let P(x_1, y_1) be the moving point and A(a, 0) B(-a, 0) are the fixed points

Given PA^2 + PB^2 = $2C^2$

$\Rightarrow$ $(x_1 - a)^2$+ $(y_1 - 0)^2$ + $(x_1 + a)^2$ + $(y_1 - 0)^2$ = $2c^2$ [using distance formula]

$\Rightarrow$ $x_1^2 + a^2 - 2x_1a + y_1^2 + x_1^2 + a^2 + 2x_1a + y_1^2 = 2c^2$

$\Rightarrow 2x_1^2 + 2y_1^2 + 2a^2 = 2c^2$

$\Rightarrow x_1^2 + y_1^2 + a^2 = c^2$

$\Rightarrow x_1^2 + y_1^2 = c^2 - a^2$

$\therefore$ Locus of (x_1, y_1) is x^2+ y^2 = c^2- a^2

20) Determine the equation of line through the point (-4, -3) and perpendicular to y-axis.

Answer : Since the line is perpendicular to y-axis, it is parallel to x-axis

$\Rightarrow$ Slope = m = 0.

Now, equation of the line is y- y_1 = m(x -x_1)

$\Rightarrow$ y + 3 = 0(x + 4)

$\Rightarrow$ y = 3 = 0

21) Find the values of k for which the line (k - 3)x-(4-k^2)y+(k^2-7k + 6) = 0 passes through the origin.

Answer : Given line (k-3)x-(4-k^2)y+(k^2-7k+6) = 0....(1)

Since the given line passes through the origin

(0, 0) must satisfy the line(1)

⇒ (k-3)0-(4-k^2)0+(k^2-7k+6) = 0

⇒ k^2- 7k + 6) = 0

⇒ (k-1)(k- 6) = 0

⇒ k = 1,6.

Hence the values of k are 1 and 6.

22) If 9x^2 + 12xy + 4y^2 + 6x + 4y - 3 = 0 represents two parallel lines, find the distance between them.

Answer : Given equation is 9x^2 + 12xy + 4y^2 + 6x + 4y - 3 = 0

⇒ $(3x+2y)^2+2(3x+2y)-3=0$

⇒ y^2+ 2y - 3 = 0 where y = 3x + 2y

⇒ (y+3) (y-1) = 0

⇒ (3x + 2y + 3) (3x + 2y - 1) = 0[y = 3x + 2y]

Hence the separate equation are 3x + 2y + 3 = 0 and 3x + 2y - 1 = 0

⇒ a = 3, b = 2, c_1 = 3 and c_2 = -1

Now, Distance between parallel lines $=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

$=\left|\frac{3-(-1)}{\sqrt{3^2+2^2}}\right|=\left|\frac{4}{\sqrt{9+4}}\right|=\left|\frac{4}{\sqrt{13}}\right|$

$=\frac{4}{\sqrt{13}}$

23) Find the equation of the line through the point of intersection of the line 5x - 6y = 1 and 3x + 2y + 5 = 0 and cutting off equal intercepts on the coordinate axis.

Answer : x +y + 2 = 0

24) Find the equation of the line through (1, 2) and which is perpendicular to the line joining (2, -3) (-1, 5)..

Answer : 3x - 8y + 13= 0

25) Find the angle between the lines 3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0.

Answer : $\tan^{-1}\left(\frac{2}{11}\right)$

26) Find the angle between the pair of straight lines given by
(a^2 - 3b^2)x^2 + 8ab xy+(b^2 -3a^2)y^2 =0.

Answer : Angle between the lines is given by tanθ=$\frac{\pm2\sqrt{h^2-ab}}{a+b}$

In this problem, tanθ =$\frac{\pm2\sqrt{16a^2b^2-(a^23b^2)(b^2-3a^2)}}{a^2-3b^2+b^2-3a^2}$

=$\frac{\pm2\sqrt{16a^2b^2-a^2b^2+3b^4+3a^4-9a^2b^2}}{-2a^2-2b^2}$

=$\frac{\pm2\sqrt{3a^4+3b^4+6a^2b^2}}{-2(a^2+b^2)}=\pm\sqrt{3}$

tanθ = 60° [If we take the acute angle]

27) Write the equation of the line through the points (1, -1) and (3, 5)

Answer : Here $x_1=1,\quad y_1=-1,\quad x_2=3,y_2=5$

Using two points form the line, we have,

$y-(-1)=\frac{5-(-1)}{3-1}(x-1)-3x+y+4=0$ Which is the required equation

28) Find the separate equations from a combined equation of a straight line $2x^2+xy-3y^2=0$

Answer : $2x^2+xy-3y^2=0$

(2x + 3y)(x - y) = 0

The separate equation of a straight line is 2x + 3y = 0 and x - y = 0.

29) Find the equation of the straight line, if the perpendicular from the origrn makes an angle of 120° with x-axis and the length of the perpendicular from the origin is 6 units.

Answer : Here P = 6 and a =120°

So the equation of the required line is of the form,

$x \cos \alpha + y \sin \alpha = p$

$x \cos 120^\circ + y \sin 120^\circ = 6$

$x \left(\frac{-1}{2}\right) + y \left(\frac{\sqrt{3}}{2}\right) = 6$

$x + \sqrt{3}y = 12$

$x - \sqrt{3}y + 12 = 0$

30) Find the slope of the straight line passing through the points (5, 7) and (7, 5).

Answer : Slope of the line m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-7}{7-5} = -1$