

QB365 Question Bank Software Study Materials

Algebra Important 2 Marks Questions With Answers (Book Back and Creative)

10th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

- 1) Reduce the rational expressions to its lowest form

$$\frac{x-3}{x^2-9}$$

Answer : $\frac{x-3}{x^2-9} = \frac{x-3}{(x+3)(x-3)} = \frac{1}{x+3}$

- 2) Find the excluded values, if any of the following expressions.

$$\frac{y}{y^2-25}$$

Answer : $\frac{y}{y^2-25} = \frac{y}{(y+5)(y-5)}$ is undefined when $(y+5)(y-5)=0$ that is $y=-5,5$.

∴ The excluded values are -5,5.

- 3) Multiply $\frac{x^3}{9y^2}$ by $\frac{27y}{x^5}$

Answer : $\frac{x^3}{9y^2} \times \frac{27y}{x^5} = \frac{3}{x^2y}$

- 4) Solve $2m^2 + 19m + 30 = 0$

Answer : $2m^2 + 19m + 30 = 2m^2 + 4m + 15m + 30 = 2m(m + 2) + 15(m + 2)$

$$= (m + 2)(2m + 15)$$

Now, equating the factors to zero we get,

$$(m + 2)(2m + 15) = 0$$

$m + 2$ gives, $m = -2$ or $2m + 15 = 0$ we get, $m = \frac{-15}{2}$

Therefore the roots are $-2, \frac{-15}{2}$

Some equations which are not quadratic can be solved by reducing them to quadratic equations by suitable substitutions. Such examples are illustrated below.

- 5) Solve the following quadratic equations by factorization method

$$4x^2 - 7x - 2 = 0$$

Answer : $4x^2 - 7x - 2 = 0$

$$4x^2 - 8x + x - 2 = 0$$

$$4x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(4x + 1) = 0$$

$$x - 2 = 0$$

$$x = 2 \text{ or } 4x + 1 = 0$$

$$x = 2, x = -\frac{1}{4}$$

- 6) Find the values of 'k', for which the quadratic equation $kx^2 - (8k + 4)x + 81 = 0$ has real and equal roots?

Answer : $kx^2 - (8k + 4)x + 81 = 0$

Since the equation has real and equal roots, $\Delta = 0$

That is, $b^2 - 4ac = 0$

Here, $a = k, b = -(8k + 4), c = 81$

That is, $[-(8k + 4)]^2 - 4(k)(81) = 0$

$$64k^2 + 64k + 16 - 324k = 0$$

$$64k^2 - 260k + 16 = 0$$

dividing by 4 we get $16k^2 - 65k + 4 = 0$

$(16k - 1)(k - 4) = 0$ then, $k = \frac{1}{16}$ or $k = 4$

- 7) Find the value(s) of 'k' for which the roots of the following equations are real and equal.

$$(5k - 6)x^2 + 2kx + 1 = 0$$

Answer : $(5k - 6)x^2 + 2kx + 1 = 0$

$$a \quad b \quad c$$

$$\Delta = b^2 - 4ac$$

$$= (2k)^2 - 4(5k-6)(1)$$

$$\Rightarrow 4k^2 - 20k + 24 = 0$$

$$\Rightarrow k^2 - 5k + 6 = 0$$

$$\Rightarrow (k-3)(k-2) = 0$$

$$k = 3, 2$$

(Since the roots are real and equal)

- 8) If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

Answer : $3x^2 + 7x - 2 = 0$ here, $a = 3, b = 7, c = -2$

since, α, β are the roots of the equation

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{3}, \alpha\beta = \frac{c}{a} = \frac{-2}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-7}{3}\right)^2 - 2\left(\frac{-2}{3}\right)}{\frac{-2}{3}} = \frac{-61}{6}$$

- 9) If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a.

Answer : Let one of the root $\alpha = 2\beta$

$$\alpha + \beta = 2\beta + \beta = 3\beta$$

Given

$$2y^2 - ay + 64 = 0$$

$$y^2 = \frac{a}{2}y + 32 = 0$$

$$\Rightarrow y^2 - \left(\frac{a}{2}\right)y + 32 = 0$$

$$\text{Sum of the roots } \alpha + \beta = \frac{a}{6}$$

$$\text{i.e } 3\beta = \frac{a}{6} \Rightarrow \beta = \frac{a}{18}$$

$$\alpha\beta = \alpha \times \frac{a}{18}$$

$$\Rightarrow 2\beta \times \beta = 2 \left(\frac{a}{18}\right) \left(\frac{a}{18}\right)$$

$$a^2 = 576$$

$$a = 24, -24$$

- 10) If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.

Answer : $3x^2 + kx + 81 = 0$

Let the roots be α and α^2

$$\alpha + \alpha^2 = \frac{-k}{3} \dots (1)$$

$$\alpha + \alpha^2 = \frac{81}{3}$$

$$\Rightarrow k^3 = 27$$

$$\Rightarrow k^3 = 3 \dots (3)$$

$$3 + 32 = \frac{-k}{3}$$

$$\Rightarrow (3 + 9)3 = -k$$

$$\Rightarrow k = -36.$$

- 11) Construct a 3 x 3 matrix whose elements are $a_{ij} = i^2j^2$

Answer : The general 3 x 3 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $a_{ij} = i^2j^2$

$$a_{11} = 1^2 \times 1^2 = 1 \times 1 = 1; a_{12} = 1^2 \times 2^2 = 1 \times 4 = 4; a_{13} = 1^2 \times 3^2 = 1 \times 9 = 9$$

$$a_{21} = 2^2 \times 1^2 = 2 \times 1 = 2; a_{22} = 2^2 \times 2^2 = 4 \times 4 = 16; a_{23} = 2^2 \times 3^2 = 4 \times 9 = 36$$

$$a_{31} = 3^2 \times 1^2 = 3 \times 1 = 3; a_{32} = 3^2 \times 2^2 = 9 \times 4 = 36; a_{33} = 3^2 \times 3^2 = 9 \times 9 = 81$$

Hence the required matrix is $A = \begin{pmatrix} 1 & 4 & 9 \\ 2 & 16 & 36 \\ 3 & 36 & 81 \end{pmatrix}$

12) If $A = \begin{bmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{bmatrix}$, find $A + B$.

Answer : It is not possible to add A and B because they have different orders.

13) Find the value of a, b, c, d, from the following matrix equation.

$$\begin{bmatrix} d & 8 \\ 3b & a \end{bmatrix} + \begin{bmatrix} 3 & a \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 2a \\ b & 4c \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix}$$

Answer : First, we add the two matrices on both left, right hand sides to get

$$\begin{bmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{bmatrix} = \begin{bmatrix} 2 & 2a+1 \\ b-5 & 4c \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have

$$d + 3 = 2 \text{ gives } d = -1$$

$$8 + a = 2a + 1 \text{ gives } a = 7$$

$$3b - 2 = b - 5 \text{ gives } b = \frac{-3}{2}$$

$$\text{Substituting } a = 7 \text{ in } a - 4 = 4c \text{ gives } c = \frac{3}{4}$$

$$\text{Therefore, } a = 7, b = \frac{-3}{2}, c = \frac{3}{4}, d = -1.$$

14) If $A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$ then verify that

$$A + B = B + A$$

Answer : L.H.S= $A+B = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \dots(1)$$

R.H.S= $B+A = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \dots(2)$$

(1)+(2) \Rightarrow L.H.S=R.H.S. Hence verified.

15) Find the order of the product matrix AB if

	(i)	(ii)	(iii)	(iv)	(v)
Orders of A	3 x 3	4 x 3	4 x 2	4 x 5	1 x 1
Orders of B	3 x 3	3 x 2	2 x 2	5 x 1	1 x 3

Answer : (i) Order of A= 3 x 3, order of B = 3 x 3

$$\text{Order of AB} = 3 \times 3$$

(ii) Order of A = 4 x 3, Order of B = 3 x 2

$$\text{Order of AB} = 4 \times 2$$

(iii) Order of A = 4 x 2, Order of B = 2 x 2

$$\text{Order of AB} = 4 \times 2$$

(iv) Order of A = 4 x 5, Order of B = 5 x 1

$$\text{Order of AB} = 4 \times 1$$

(v) Order of A = 1 x 1, Order of B = 1 x 3

$$\text{Order of AB} = 1 \times 3$$

16) Write down the quadratic equation in general form for which sum and product of the roots are given below.

$$-\frac{7}{2}, \frac{5}{2}$$

Answer : $x^2 - \left(-\frac{7}{2}\right)x + \frac{5}{2} = 0$ gives $2x^2 + 7x + 5 = 0$

17) Determine the quadratic equations, whose sum and product of roots are

$$\frac{5}{3}, 4$$

Answer : Sum of the roots = $\frac{5}{3}$

Product of the roots = 4

General form of the Quadratic equation is $x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$

$$\Rightarrow x^2 - \frac{5}{3}x + 4 = 0$$

Simplifying $3x^2 - 5x + 12 = 0$

- 18) Find the sum and product of the roots for each of the following quadratic equations

$$x^2 + 3x = 0$$

Answer : $x^2 + 3x = 0$

Comparing with $ax^2 + bx + c = 0$

$a = 1, b = 3, c = 0$

Sum of the roots $-\frac{b}{a} = -\frac{3}{1} = -3$

Products of the roots $\frac{c}{a} = \frac{0}{1} = 0$

- 19) If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$$

Answer : $x^2 + 7x + 10$ here, $a = -1, b = 7, c = 10$

if α and β are roots of the equation then,

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7; \alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$$

$$\begin{aligned} \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} \\ &= \frac{(-7)^3 - 3(10)(-7)}{10} = \frac{-343 + 210}{10} = \frac{-133}{10} \end{aligned}$$

- 20) Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.

$$\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$$

Answer : $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha} = \frac{\beta + \alpha}{\alpha^2\beta^2}$
 $= \frac{\alpha + \beta}{(\alpha\beta)^2}$

- 21) Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.

$$\frac{\alpha + 3}{\beta} + \frac{\beta + 3}{\alpha}$$

Answer : $\frac{\alpha + 3}{\beta} + \frac{\beta + 3}{\alpha} = \frac{\alpha^2 + 3\alpha + \beta^2 + 3\beta}{\alpha\beta}$
 $= \frac{\alpha^2 + \beta^2 + 3(\alpha + \beta)}{\alpha\beta}$
 $= \frac{(\alpha + \beta)^2 - 2\alpha\beta + 3(\alpha + \beta)}{\alpha\beta}$

- 22) The roots of the equation $x^2 + 6x - 4 = 0$ are α, β . Find the quadratic equation whose roots are

$$\frac{2}{\alpha} \text{ and } \frac{2}{\beta}$$

Answer : If the roots are given, the quadratic equation is $X^2 - (\text{sum of the roots})x + \text{product the roots} = 0$. For the given equation.

$$x^2 - 6x - 4 = 0$$

$$\alpha + \beta = -6$$

$$\alpha\beta = -4$$

$$\frac{2}{\alpha} \times \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta} = \frac{2(-6)}{-4}$$

$$= \frac{-12}{-4} = 3$$

$$\frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{-4} = -1$$

\therefore The required equation is $x^2 - 3x - 1 = 0$

- 23) If $A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$ then verify that

$$A + (-A) = (-A) + A = 0$$

Answer : $A + (-A) = (-A) + A = 0$

L.H.S = $A+(-A)$

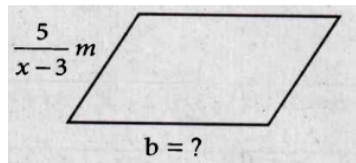
$$= \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(1)$$

R.H.S = $(-A)+A$

$$= \begin{bmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(2)$$

(1) = (2) \Rightarrow L.H.S. = R.H.S. Hence verified.

- 24) Find the base of the given parallelogram whose perimeter is $= \frac{4x^2+10x-50}{(x-3)(x+5)}$



Answer : $Perimeter = \frac{4x^2+10x-50}{(x-3)(x+5)}$

$$2(l + b) = \frac{4x^2+10x-50}{(x-3)(x+5)}$$

$$2\left(\frac{5}{x-3} + b\right) = \frac{2(2x-5)(x+5)}{(x-3)(x+5)}$$

$$b = \frac{2x-5}{x-3} - \frac{5}{x-3}$$

$$= \frac{2x-5-5}{x-3} = \frac{2x-10}{x-3}$$

- 25) Find the element in the second row and third column of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$

Answer : $a_{23} = 25$

- 26) Find the values of k for which the following equation has equal roots.

$$(k - 12)r + 2(k - 12)x + 2 = 0$$

Answer : $\frac{(k-12)}{a}x^2 + \frac{2(k-12)}{b}x + \frac{2}{c} = 0$

$$D^2 = b^2 - 4ac = (2(k - 12))^2 - 4(k - 12)(2)$$

$$= 4(k - 12)[(k - 12) - 2]$$

$$= 4(k-12)(k- 14)$$

The given equation will have equal roots, if $D = 0$

$$\Rightarrow 4(k-12)(k-14) = 0$$

$$k - 12 = 0 \text{ or } k - 14 = 0$$

$$k = 12, 14$$

- 27) Simplify : $\frac{x^2+3x}{x^2-4x-21}$

Answer : $\frac{x^2+3x}{x^2-4x-21} = \frac{x(x+3)}{(x-7)(x+3)}$

$$= \frac{x}{x-7}$$

- 28) Find the Quadratic equation whose sum and product of the roots are (i) - 12, 32 (ii) $2a, a^2-b^2$.

Answer : (i) Sum = -12,

Product = 32.

Quadratic equation:

$$x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0$$

$$x^2 + 12x + 32 = 0$$

(ii) Sum of the roots ' $2a$ '

Product of the roots ' $a^2 - b^2$ '

Quadratic equation:

$$x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0$$

$$x^2 - 2ax + a^2 - b^2 = 0$$

- 29) Find the transpose of $A = \begin{bmatrix} 0 & 0 \\ 3 & 8 \\ 8 & 7 \end{bmatrix}$

Answer : $A = \begin{bmatrix} 0 & 0 \\ 3 & 8 \\ 8 & 7 \end{bmatrix}$
 $\therefore A^T = \begin{bmatrix} 0 & 3 & 8 \\ 0 & 8 & 7 \end{bmatrix}$

30) If $\begin{bmatrix} x - y & 2y \\ 2y + z & x + y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$, then find the value of $\mathbf{x + y + z}$

Answer : Since the matrices are equal,

Let us equate the corresponding elements.

$$\text{Now, } x - y = 1, 2y = 4$$

$$2y + z = 9, x + y = 5$$

$$2y - 4, x + y = 5 \quad 2y + z = 9$$

$$y = 2 \quad x + 2 = 5 \quad 2(2) + z = 9$$

$$x = 3 \quad z = 9 - 4 = 5$$

$$\therefore x + y + z = 3 + 2 + 5 = 10$$