QB365 Question Bank Software Study Materials

Geometry Important 2 Marks Questions With Answers (Book Back and Creative)

10th Standard

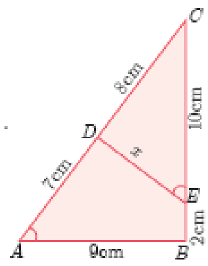
Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) $\angle A=\angle CED$ prove that $\Delta\ CAB\sim\Delta CED$ Also find the value of x.



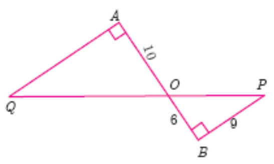
Answer : $\Delta\ CAB$ and ΔCED,$\angle C$ is common,$\angle A=\angle CED$

Therefore,$\Delta CAB\sim\Delta CED$

Hence,$\frac{CA}{CE}=\frac{AB}{DE}=\frac{CB}{CD}$

$\frac{AB}{DE}=\frac{CB}{CD}$ $\frac{9}{x}=\frac{10+2}{8},x=\frac{8\times9}{12}=6$ cm.

2) QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



Answer : ΔAOQ and $\Delta BOP,\angle OAQ=\angle OBP=90^0$

$\angle AOQ=\angle BOP$ (Vertically opposite angles)

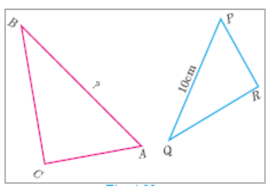
Therefore, by AA Criterion of similarity,

$\Delta AOQ\sim\Delta BOP$

$\frac{AO}{BO}=\frac{OQ}{OP}=\frac{AQ}{BP}$

$\frac{10}{6}=\frac{AQ}{9}$ gives $AQ=\frac{10\times9}{6}=15cm$

3) The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB



Answer : The ratio of the corresponding sides of similar triangles is same as the ratio of their perimeters.

Since, $\Delta ABC\sim\Delta PQR$

$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{36}{24}$

$\frac{AB}{PQ}=\frac{36}{24}$ $\frac{AB}{10}=\frac{36}{24}$

$AB=\frac{36\times10}{24}=15cm$

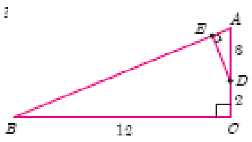
4) If $\triangle$ABC is similar to$\triangle$DEF such that BC = 3 cm, EF = 4 cm and area of $\triangle$ABC = 54 cm^2. Find the area of $\triangle$DEF.

Answer : Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)}=\frac{BC^2}{EF^2}$ gives $\frac{54}{Area(\Delta DEF)}=\frac{3^2}{4^2}$

$Area(\Delta DEF)=\frac{16\times54}{9}=96cm^2$

5) In the adjacent figure, $\triangle$ABCis right angled at C and DE$\perp$ AB. Prove that $\triangle$ABC~$\triangle$ADE and hence find the lengths of AE and DE.



Answer : Given < C = 90° = < DEA

< A is common to both the triangles $\triangle$ABC and $\triangle$ADE

By AA criteria for similarity

$\triangle ABC \sim \triangle ADE$

Their corresponding sides are proportional

$\therefore \frac{BC}{DE} = \frac{AC}{AE} = \frac{AB}{AD}$

$\frac{12}{DE} = \frac{3+2}{AE} = \frac{AB}{3}$

Also in right ABC , < C = 90°

using Pythagoras theorem, we have

$AB^2 = BC^2 + AC^2 = 12^2 + 5^2$

= 144 + 25 = 169

AB = 13 cm

Now from (1), we get

$\frac{12}{DE} = \frac{13}{3}$

$\mathrm{DE} = \frac{12\times3}{13} = \frac{36}{13}$

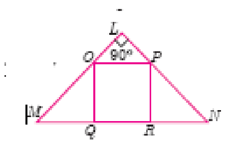
DE = 2.77 cm

Also from (1) $\frac{5}{AE} = \frac{13}{3}$

$\mathrm{AE} = \frac{5\times3}{13} = \frac{15}{13} = 1.15 \text{ cm}$

6) If figure OPRQ is a square and ∠MLN = 90°. Prove that

$\triangle$LOP ~$\triangle$QMO



Answer : In ΔLOP & ΔQMO,

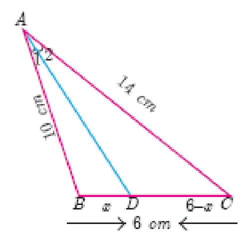
∠OLP = ∠MQO 90°

and ∠LOP =∠OMQ (corresponding angles)

(by AA criterion of similarity)

ΔLOP ~ ΔQMO

7) In the Figure, AD is the bisector of ∠BAC, if A = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.



Answer : Let BD = x cm, then DC = (6 – x)cm

AD is bisector of∠ A

Therefore by Angle Bisector Theorem

$\frac{AB}{AC} = \frac{BD}{DC}$

$\frac{10}{14} = \frac{x}{6-x} \quad \frac{5}{7} = \frac{x}{6-x}$

So, 12x = 30 we get, $x = \frac{30}{12} = 2.5$

Therefore, BD = 2.5 cm, DC = 6−x = 6−2.5 = 3.5 cm

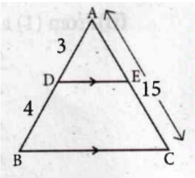
8) In $\triangle$ABC,D and E are points on the sides AB and AC respectively such that DE||BC $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15cm find AE.

Answer : Given $\frac{AD}{DB} = \frac{3}{4}$

AC = 15 cm

EC = AC - AE

= 15 - AE



By Basic proportionality theorem

We have $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{3}{4} = \frac{AE}{AC-AE}$

$\frac{3}{4} = \frac{AE}{15-AE}$

3(15 - AE) = 4AE

45 - 3AE = 4AE

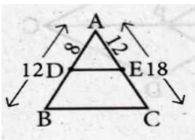
45 = 4AE + 3AE

7AE = 45

$\text{AE} = \frac{45}{7} = 6.43 \text{ cm}$

9) In $\triangle$ABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE||BC

AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

Answer : 

Given AB = 12 cm, AD = 8 cm, AE = 12 cm, AC = 18 cm

By basic proportionality theorem $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{AD}{DB} = \frac{8}{AB-AD} = \frac{8}{12-8}$

$\frac{AD}{DB} = \frac{8}{4} = 2$

$\frac{AE}{EC} = \frac{12}{AC-AE}$

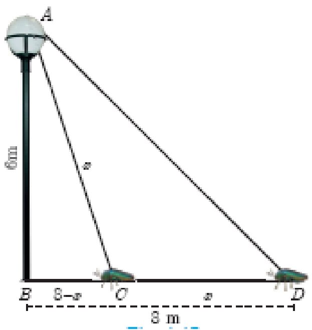
$\frac{AE}{EC} = \frac{12}{18-12} = \frac{12}{6} = 2$

From (1) and (2)

$\frac{AD}{DB} = \frac{AE}{EC}$

DE ll BC

10) An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?

Answer : 

Distance between the insect and the foot of the lamp post BD = 8 m

The height of the lamp post, AB = 6 m

Aft er moving a distance of x m, let the insect be at C

Let, AC = CD = x . Then BC = BD − CD = 8 − x

In $\triangle$ABC, $\angle$B = 90°

$AC^2 = AB^2 + BC^2$ gives x^2 = 62 + (8 - x)2

$x^2 = 36 + 64 - 16x + x^2$

16x = 100 then x = 6.25

Then, BC = 8 − x = 8 − 6.25 = 1.75m

Therefore the insect is 1.75 m away from the foot of the lamp post.

11) In the rectangle WXYZ, XY+YZ = 17 cm, and XZ + YW = 26 cm .Calculate the length and breadth of the rectangle

Answer : XY + CZ = 17cm

XZ + YW = 26cm

We know that diagonals if a rectangle bisect each other and the diagonals have equal length.

$\therefore$ Each diagonal $= \frac{26}{2} = 13$ cm

i.e., XZ = 13 cm and YW = 13 cm

Also given XY + yZ = 17 cm

Squaring on both sides $(XY + YZ)^2 = 17^2$

$(XY)^2 + (YZ)^2 + 2 \times (XY) \times (YZ) = 289$

By Pythagoras theorem $(XY)^2 + (YZ)^2 = XZ^2$

$\therefore [XZ]^2 + 2(XY) \times (YZ) = 289$

132 + 2 xl ength x breadth = 289

2 x Area = 289 - 169

Area $= \frac{289-169}{2} = \frac{120}{2}$

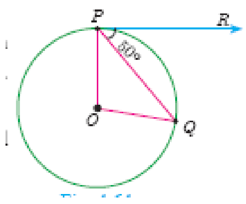
The possible length and breadth are

(1,60) (2,30) (3,20) (4, 15), (5, 12) (6, 10).

In this pair the length and breadth should satisff pythagoras theorem for diagonal.

5,12 is the possible length and breadth.

12) In Figure, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ,



Answer : ∠OPQ = 90° - 50° = 40° (angle between the radius and tangent is 90°)

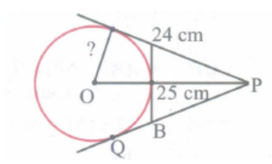
OP = OQ (Radii of a circle are equal)

∠OPQ = ∠OQP = 40° (△OPQ is isosceles)

$\angle POQ = 180^0 - \angle OPQ - \angle OQP$

∠POQ = 180° - 40°- 40° = 100°.

13) The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

Answer : 

Let 'O' be the center of the circle AP be the tangent.

Given OA = 25 cm; AP = 24 cm. Op is the radius.

Tangent and radius through the point are perpendicular to each other.

In the right triangle △OPA,

$OA^2 = OP^2 + PA^2$

$25^2 = OP^2 + 24^2$

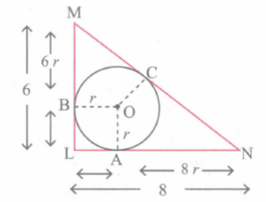
$625 = OP^2 + 576$

$OP^2 = 625 - 576 = 49$

OP = 7cm

Radius = 7cm

14) △ LMN is a right angled triangle with∠L = 90°. A circle is inscribed in it. The lengths of the sidescontaining the right angle are 6 cm and 8 cm. Find the radius of the circle.

Answer : 

Given △LMN is a right angled triangle with ∠L = 90°

By Pythagoras theorem,

$Since PL \| \mathrm{OQ}, \mathrm{PL}=\mathrm{r}=\mathrm{OP}$

we have $\mathrm{NR}=\mathrm{NP}=\mathrm{NL}-\mathrm{PL}$

[NR and NP are tangents]

= (6 - r)cm

MR = MQ = ML- LQ = (8 - r) cm

[MR and MQ are tangents]

NM = NR+ RM

= (6 - r + 8 - r)cm

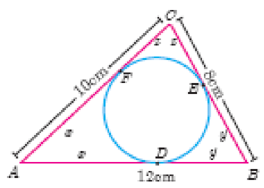
= (14 - 2r) cm

Now $NM^2 = NL^2 + LM^2$ [By pythagoras Theorem]

(14-2r)2 = 82+62

(14-2r)2 = 64+36

15) A circle is inscribed in △ABC having sides 8 cm, 10 cm and 12 cm as shown in figure, find AD, BE and CF.



Answer : We know that the tangents drawn from are external point to a circle are equal.

Therefore AD AF = x

BD = BE = y

and CE = CF = z

Now, AB = 12 cm, BC = 8 cm, and CA = 10 cm.

x + y = 12,y + z = 8 and z + x = 10

(x + y) + (y + z) + (z + x) = 12 + 8 + 10

2(x + y + z) = 30

x+ y + z = 15

Now, x + y = 12 and x + y + z = 15

12 + z = 15

Z = 3

y + z = 8 and x + y + z = 15

$x+8=15 \Rightarrow x=7$

and z + x = 10 and x + y + z = 15

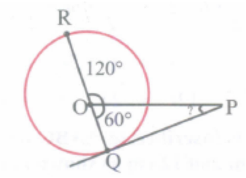
$10+y=15 \Rightarrow y=5$

Hence, AD = x = 7cm,

BE = y = 5 cm and

CF = z = 3 cm

16) PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of thecircle such that ∠PQR = 120°. Find ∠OPQ.

Answer : 

Given PQ is the tangent from the point P outside the circle and OQ is the radius.

We know that tangent meet the radius perpendicularly

$\therefore \angle PQO = 90^\circ$

$\angle POQ = 180 - 120 = 60^\circ$

$[\because\ \angle\ POQ\ and\ \angle POR\ are\ linear\ pair\ of\ angles]$

$In\ \triangle POO \angle POO + \angle POO + \angle OPQ = 180^\circ$

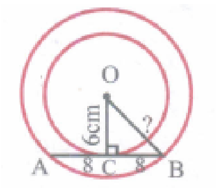
[sum of angles of a triangle]

$60^o + 90^o + \angle OPQ = 180o$

$\angle OPQ = 180^o - 150^o$

$\angle OPQ = 30^o$

17) In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Answer : 

Let o be the center of concentric circies and APB be the chord of length 16 cm of the larger circle touching the smaller circle at p.

Then OP $\perp$ AB and p is the midpoint of AB.

AP = PB = 8 cm

In LOPA, we have

$OA^2 = OP^2 + AP^2$ [By pythagoras Theorem]

$OA^2 = 6^2 + 8^2$

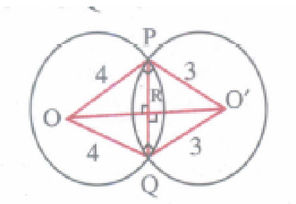
$OA^2 = 36 + 64$

$OA^2 = 100$

OA = 10cm

Radius of the larger circle in 10 cm

18) Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Answer : 

Since the tangents at a point to a circle is

perpendicular to the radius through the point of contact

$\therefore \angle OPO' = 90^{\circ}$

OP^2 + OP^2 = (OO')^2

[By Pythagoras theorem]

3^2 + 4^2 = (OO')^2

9+16 = (OO')^2

25 = (OO')^2

OO' = 5cm

Since the line joining the centres of two intersecting circles is perpendicular bisector of their common chord.

$\mathrm{OR} \perp \mathrm{PQ}$ and $O'\mathrm{R} \perp \mathrm{PQ}$

AIso PR = QR

Let OR = x, then O'R = 5 - x

AIso at PR = QR = y cm

In $\triangle ORP$ and $\Delta O' RP$

Applying pythagoras theorem

OP^2 = OR^3 + RP^2 and O'P'^{2} = O'R^2 + RP^2

$3^2 = x^2 + y^2 and 4^2 = (5-x)^2 + y_i^2$

$Subtracting \Rightarrow 4^2 - 3^2 = \left\{(5-x)^2 + y^2\right\} - \left(x^2 + y^2\right)$

$16 - 9 = 25 - 10x + x^2 + y^2 - x^2 - y^2$

7 - 25 = 10x

10x = 25 - 7

10x = 18

x = 1.8 cm

3^2 = x^2 + y^2

$y = \sqrt{9 - (1.8)^2} = \sqrt{5.76}$

y = 2.4cm

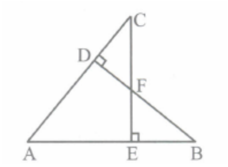
Hence PR = QR = 2.4 cm

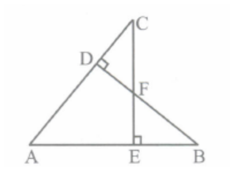
PQ = 2y = 4.8 cm

19) In the figure, if BD⊥AC and CE $\perp$ AB, prove that

(i) $\Delta AEC \sim \Delta ADB$

(ii) $\frac{CA}{AB} = \frac{CE}{DB}$



Answer : 

$\Delta AEC \quad \Delta ADB$

$\angle AEC = \angle ADB = 90^0$

$\angle CAE = \angle BAD$

[common By AA similarity criteria]

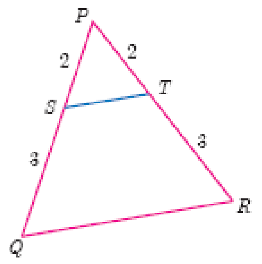
$\Delta AEC \sim \Delta ADB$

(ii) Their corresponding sides are Proportional

$\frac{CA}{AB} = \frac{CE}{DB}$

Hence proved.

20) Show that △PST~△PQR



Answer : In △PST and △PQR

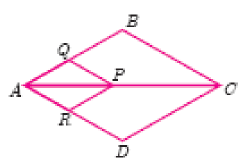
$\frac{PS}{PQ} = \frac{2}{2+3} = \frac{2}{5}, \frac{PT}{PR} = \frac{2}{2+3} = \frac{2}{5}$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and ∠P is is common

Therefore, by SAS similarity,

△PST~△PQR

21) In fig. if PQ || BC and PR || CD prove that



$\frac{QB}{AQ} = \frac{DR}{AR}$

Answer : From (1) and (2) we have

$\frac{AQ}{AB} = \frac{AR}{AD}$

$\frac{AB}{AQ} = \frac{AD}{AR}$

$\frac{AQ+QB}{AQ} = \frac{AR+RD}{AR}$

$1 + \frac{QB}{AQ} = 1 + \frac{RD}{AR}$

$\Rightarrow \frac{QB}{AQ} = \frac{DR}{AR}$

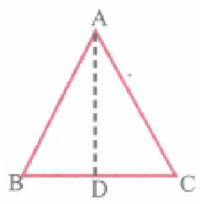
22) Check whether AD is bisector ∠A of △ABC in each of the following AB = 4cm, AC = 6cm, BD = 1.6cm and CD = 2.4cm.

Answer : AB = 4 cm,

AC = 6 crn,

BD = 1.6 cm,

CD = 2.4 cm.



$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$

$\frac{BD}{DC} = \frac{1.6}{2.4} = \frac{2}{3}$

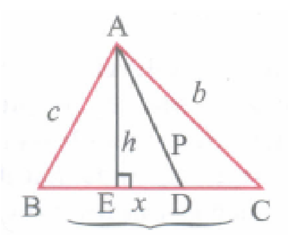
$\frac{AB}{AC} = \frac{BD}{DC}$

By the converse of the Angle Bisector theorem AD is the bisector of ∠A

23) D is the mid point of side BC and AE ⊥ BC. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that

$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Answer :



From (i) and (ii) we get

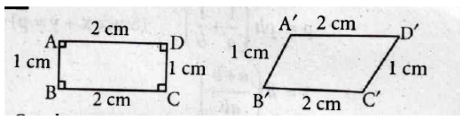
$AC^2 + AB^2 = AD^2 + BC \cdot DE + \frac{1}{4}BC^2 + AD^2 - BC \cdot DE + \frac{BC^2}{4}$

$= 2AD^2 + 2\left(\frac{BC^2}{4}\right)$

$AC^2 + AB^2 = 2AD^2 + \frac{BC^2}{2}$

$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

24) Are a rectangle and a parallelogram similar. Discuss.

Answer : 

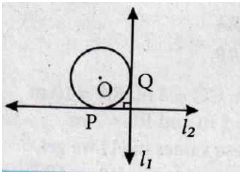
On observing the given figures.

Their corresponding sides are proportional but their corresponding angles are not equal.

The shapes parallelogram and rectangle are not similar.

25) Can we draw two tangents perpendicular to each other on a circle?

Answer : Yes. we can draw two tangents perpendicular to each other.



26) Check the given sides are the sides of a right angled triangle.

(i) a = 6 cm,b = 5 cm and c = 10 cm

(ii) a = 5 cm b = 5 cm and c = 11 cm

Answer : (i) We have a = 6cm,

b = 5 cm and c = 10cm

Here the larger side is c = 10 cm

Threfore $a^2 + b^2 = 6^2 + 8^2$

= 36 + 64 = 100

= $(10)^2 = C^2$

Therefore The triangle with the given sides is a right triangle.

(ii) We have a = 5 cm,

b = 8 cm and c = 11 cm

Here the larger side is c = 11 cm

$a^2 + b^2 = 5^2 + 8^2 = 25 + 64$

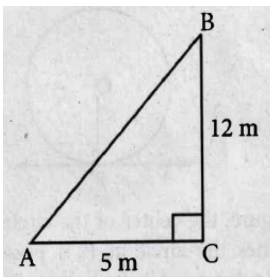
$a^2 + b^2 = 89$

But $C^2 = 11^2 = 121$

$\therefore a^2 + b^2 \neq c^2$

The triangle with the given sides are not a right angled triangle.

27) A ladder is placed in such a way that its foot is at a distance of 5 m from a wall and its tip reaches a window 12 m above the grouud. Determine the length of the ladder.

Answer : 

Let AB be the ladder and B be the window, then

BC = 12 m and AC = 5m

Since ΔABC is right triangle; right angled at C

$AB^2 = AC^2+BC^2$

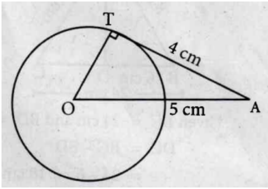
$AB^2 = 5^2+12^2 = 25+144$

$AB^2 = 169$

AB = 13m

Hence the lengh of the ladder is 13 m

28) The length of a tangent from a point ar a distance of 5 cm from the center of the circle is 4 cm. Find the radius of the circle

Answer : 

The tangent to a circle is perpendicular to the radius through the point of contact

$\angle OTA = 90^{o}$

Now in right ΔOTA

$OA^2 = OT^2 + TA^2$

$5^2 = OT^2 + 4^2$

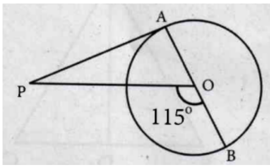
$OT^2 = 5^2 - 4^2$

$= 25 - 16 = 9$

$OT = 3$

Thus the radius of the circle is 3 cm.

29) In the figure PA is a tangent from an external point P to a circle with cenfre O. If $\angle POB = 115^{\circ}$ then find $\angle APO$

Answer : 

Here PA is a tangent and oA is radius. Also a radius through the point of contact is perpendicular to the tangent.

$OA \perp PA$

$\angle PAO = 90^{\circ}$

In $\triangle OAP, \angle POB$ in an external angle.

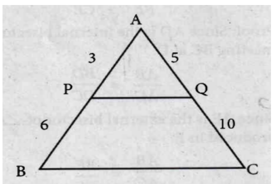
$\therefore \angle APO + \angle PAO = \angle POB$

$\angle APO + 90^{\circ} = 115^{\circ}$

$\angle APO = 115^{\circ} - 90^{\circ} = 25^{\circ}$

$\angle APO = 25^{\circ}$

30) P and Q are points on sides AB and AC respectively of ΔABC. If AP = 3 cm, PB = 6 cm, AQ = 5 cm and QC = 10 cm. Show that BC = 3PQ.

Answer : 

Thales Theorem,

$\frac{AP}{PB} = \frac{3}{9} = \frac{1}{3}$

$\frac{AQ}{QC} = \frac{5}{15} = \frac{1}{3}$

$\therefore \triangle APQ \sim \triangle ABC$

(By SAS similarity criterion)

$\therefore \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow \frac{PQ}{BC} = \frac{1}{3} = BC = 3PQ$