QB365 Question Bank Software Study Materials

Numbers and Sequences Important 2 Marks Questions With Answers (Book Back and Creative)

10th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Answer : All the integers 'a' must be either even or odd.

If it is even then a = 2q.

If it is odd then a = 2q + 1

Case 1:

If a = 2q

$a^2 = (2q)^2$

$a^2 = 4q^2$, remainder 0 when divided by 4.

Case 2:

If a = 2q+ 1

$a^2 = (2q + 2)^2$

$= 4q^2 + 4q + 1$

= 4q (q + 1) + 1

$a^2 = 4m + 1$ Where m = q (q + 1) is an integer

It is of the form bq + 1 where 1 is the remainder when divided by 4.

The square of any integer leaves the remainder either 0 or 1 when divided by 4.

2) In the given factorisation, find the numbers m and n.

Answer : Value of the first box from bottom = 5 x 2 = 10

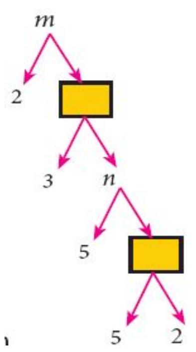
Value of n = 5 x 10 = 50

Value of the second box from bottom = 3 x 50 = 150

Value of m = 2 x 150 = 300

Thus, the required numbers are

m = 300, n = 50



3) Determine the value of d such that 15 $\equiv$ 3 (mod d).

Answer : 15 $\equiv$ 3 (mod d) means 15 - 3 = kd, for some integer k,

12 = kd

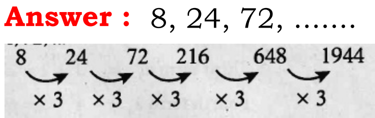
gives d divides 12.

The divisors of 12 are 1,2,3,4,6,12. But d should be larger than 3 and so the possible values for d are 4, 6, 12.

4) Find the next three terms of the following sequence.

8, 24, 72, ...

Answer : 8, 24, 72, ...



Each term is obtained by multiplying the previous term by 3.

Next three terms are 216, 648, 1944.

5) Find the n^{th} term of the following sequences,

2, 5, 10, 17,....,

Answer : 2, 5, 10, 17,....,

= $1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1$ ...

Here the every term is obtained by adding 1 to its square

The general term $a_n = n^2 + 1$

6) Find the number of terms in the A.P. 3, 6, 9, 12,..., 111.

Answer : First term a = 3; common difference d = 6 - 3 = 3 ; last term l = 111

We know that, n = $\left(\frac{l-a}{d}\right)+1$

n = $\left(\frac{111-3}{3}\right)+1$ = 37

Thus the A.P. contain 37 terms

7) Find the 8^{th} term of the G.P 9,3,1,....

Answer : The find the 8th term we have use the n^{th} term formula $tn = ar^{n-1}$

First term a = 9, common radio r = $\frac{t_2}{t_1}=\frac{3}{9}=\frac{1}{3}$

$t_8 = 9\times\left(\frac{1}{3}\right)^{8-1} = 9\times\left(\frac{1}{3}\right)^7 = \frac{1}{243}$

Therefore the 8th term of the G.P is $\frac{1}{243}$

8) If a, b, c are in A.P. then show that 3^a, 3^b, 3^c are in G.P

Answer : Given a, b, c are in A.P

$t_2 - t_1 = t_3 - t_2$

b - a = c - b

b + b = c + a

2b = c + a

If we multiply both the sides by same number value will not change

$3^{2b} = 3^{a+c}$

$3^{b+b} = 3^{a+c}$

$3^b . 3^b = 3^a . 3^c$

$\frac{3^b}{3^a}=\frac{3^c}{3^b}$

Thus 3^a, 3^b, 3^c are in G.P.

9) Find the first term of a G.P. in which S_6 = 4095 and r = 4

Answer : Common ratio = 4 > 1, sum of first 6 terms S_6 = 4095

Hence , S_6 = $\frac{a(r^n-1)}{r-1} = 4095$

Since, r = 4, $\frac{a(4^6-1)}{4-1}$ = 4095 gives a x $\frac{4095}{3} = 4095$

First term a = 3.

10) Find the sum 3 + 1+ $\frac{1}{3}$ + ...∞

Answer : Here a = 3, r = $\frac{t_2}{t_1}=\frac{1}{3}$

Sum of infinite terms = $\frac{a}{1-r}=\frac{3}{1-\frac{1}{3}}=\frac{9}{2}$

11) If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio

Answer : First term a = 8

$S_\propto = \cdot\frac{32}{3}$

Sum upto infinity of a G.P.,

$S_\propto = \frac{a}{1-r}$

$\frac{32}{3}=\frac{8}{1-r}$

32(1 - r) = 24

$1-r=\frac{24}{32}$

$1-r=\frac{3}{4}$

$1-\frac{3}{4}=r$

$\frac{1}{4}=r$

Common ratio $r=\frac{1}{4}$

12) Find the quotient and remainder when a is divided by b in the following a = 17 b = -3

Answer : a = 17 b = -3

By Euclid's division lemma

a = bq + r , Where $0 \leq r < |b|$

17 =(-3) x (-5) + 2 $0 \leq r < |-3|$

Therefore Quotient q = - 5

Remainder r = 2

13) Find the least positive value of x such that

$5x \equiv 4 \pmod 6$

Answer : $5x \equiv 4 \pmod 6$

5x - 4 = 6n for some integer n.

5x - 4 is a multiple of 6.

x = 2 is the least positive x.

14) Find the next three terms of the sequences.

1, 0.1, 0.01,...

Answer : 1, 0.1, 0.01, ...
÷10 ÷10

Here each term is divided by 10. Hence , the next three terms are

$a_4 = \frac{0.01}{10} = 0.001$

$a_5 = \frac{0.001}{10} = 0.0001$

$a_6 = \frac{0.0001}{10} = 0.00001$

15) Find the first four terms of the sequences whose nth terms are given by

$a_n = 2n^2 - 6$

Answer : $a_n = 2n^2 - 6$

n^{th} term is given by $a_n = 2n^2 - 6$

First term $a_1 = 2(1)^2 - 6 = -4$

Second term $a_2 = 2(2)^2 - 6 = 2$

Third term $a_3 = 2(3)^2 - 6 = 12$

Fourth term $a_4 = 2(4)^2 - 6 = 26$

∴ The first four terms are -4, 2, 12, 26, ...

16) Check whether the following sequences are in A.P. or not ?

2, 4, 8, 16,........

Answer : $t_2 - t_1 = 4 - 2 = 2$

$t_3 - t_2 = 8 - 4 = 4$

$t_2 - t_1 \neq t_3 - t_2$

Thus, the differences between consecutive terms are not equal. Hence the terms of the sequence 2, 4, 8, 16, . . . are not in A.P.

17) Check whether the following sequences are in A.P.

9, 13, 17, 21, 25,... ,

Answer : 9,13,17, 21, 25, ...

$t_2 - t_1 = 13 - 9 = 4$

$t_3 - t_2 = 17 - 13 = 4$

$t_4 - t_3 = 21 - 17 = 4$

$t_5 - t_4 = 25 - 21 = 4$

$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = t_5 - t_4 = 4$

The difference between consecutive terms are equal.

18) Check whether the following sequences are in A.P.

$\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, .$

Answer : $t_2 - t_1 = 0 - \left(\frac{-1}{3}\right) = \frac{1}{3}$
$t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$
$t_4 - t_3 = \frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$
$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \frac{1}{3}$
The differences between consecutive terms are equal.
$\therefore \frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$ are in A P

19) Which of the following sequences form a Geometric Progression?
5, 25, 50, 75

Answer : 5, 25, 50, 75,....,
$\frac{t_2}{t_1} = \frac{25}{5} = 5; \frac{t_3}{t_2} = \frac{50}{25} = 2; \frac{t_4}{t_3} = \frac{75}{50} = \frac{3}{2}$
Since the ratios between successive terms are not equal, the sequence 5, 25, 50, 75,... is not a Geometric Progression.

20) Find the geometric progression whose first term and common ratios are given by
a = 256 , r = 0.5

Answer : The general form of Geometric progression is a, ar, ar^2,...
a = 256, ar = 256 x 0.5 = 128 , ar^2 = 256 x $(0.5)^2$ = 64
Therefore the required Geometric progression is 256,128, 64,....

21) Which of the following sequences are in G.P.?
$\frac{1}{3}, \frac{1}{6}, \frac{1}{12},$

Answer : $\frac{t_2}{t_1} = \frac{1/6}{1/3} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$
$\frac{t_3}{t_2} = \frac{1/12}{1/6} = \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$
The ratios between successive terms are equal $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}$ are in G.P.

22) Find the sum of
1 + 3 + 5 +..+ 55

Answer : 1 + 3 + 5 +...+ 55
Here the number of terms is not given.
Now we have to find the number of terms using the formula, $n = \frac{(l-a)}{d} + 1$ gives n = $\frac{(55-1)}{2} + 1 = 28$
Therefore, 1 + 3 + 5 +..+ 55 = $(28)^2$ = 784

23) Is 1 a prime number ?

Answer : No, 1 is not a prime number.
Because the definition of a prime number is "a positive integer that has exactly two positive divisors (1 and itself) is a prime number''. But 1 has only one positive divisor 1 itself.
So it is not a prime.

24) What is the sum of first n even natural numbers?

Answer : Sum of n numbers $S_n = \frac{n}{2}[2a + (n-1)d]$
Let the first n even natural numbers be 2, 4, 6, 8, ... upto n terms.
a = 2; d = 4 - 2 = 2
$S_n = \frac{n}{2}[2(2) + (n-1)(2)]$
$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2]$
$= \frac{n}{2} \times 2(n+1) = n(n+1)$
Sum of first n even natural numbers = n (n + 1)

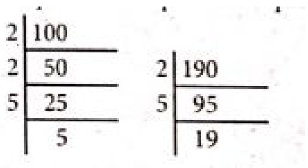
25) Split 64 into three parts such that the numbers are in G.P

Answer : Let the three numbers be ar^{n-1}, a, ar
Let a = 4 and r = 4
The three numbers are 1,4,16

26) Find the H.C.F. and L.C.M of 100 and 190 by fundamental theorem of arithmetic

Answer : By fundamental theorem we have every composite number can be expressed as a product of primes

Factorizing 100 and 190



$100 = 2^2 \times 5^2$

$190 = 2^1 \times 5^1 \times 19^1$

$\therefore HCF$ of 100 and 190 $= 2^1 \times 5^1 = 10$

H.C.F. $\times$ L.C.M. $=$ Product of two numbers

$10 \times$ L.C.M. $= 100 \times 190$

L.C.M. $= \frac{100 \times 190}{10} = 1900$

27) Does 7 divides $\left(2^{29} + 3\right)$?

Answer : We have

$2^3 \equiv 1(\bmod 7)$

$\left(2^3\right)^8 \equiv 1^8(\bmod 7)$

$2^3 2^{24} \equiv 1 \times 1^8(\bmod 7)$

$2^{27} \equiv 1(\bmod 7)$

$2^2 2^{27} \equiv 4 \times 1(1 \bmod 7)$

$2^{29} \equiv 4(\bmod 7)$

$2^{29} + 3 = (4 + 3)(\bmod 7)$

$2^{29} + 3 \equiv 0(\bmod 7)$

$2^{29} + 3$ is divisible by 7

28) Find the first three terms of $a_n = \frac{2n-3}{6}$

Answer : Given $a_n = \frac{2n-3}{6}$

$\therefore a_1 = \frac{2(1)-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$

$a_2 = \frac{2(2)-3}{6} = \frac{4-3}{6} = \frac{1}{6}$

$a_3 = \frac{2(3)-3}{6} = \frac{6-3}{6} = \frac{3}{6}$

$\therefore$ First three terms are $\frac{-1}{6}, \frac{1}{6}, \frac{3}{6}$.

29) Which term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is the first negative term?

Answer : $t_2 - t_1 = 19\frac{1}{4} - 20 = -\frac{3}{4}$

$t_3 - t_2 = 18\frac{1}{2} - 19\frac{1}{4} = -\frac{3}{4}$

$t_2 - t_1 = t_3 - t_2$

$\therefore$ The sequence is an A.P. with $a = 20; d = -\frac{3}{4}$

Let the n^{th} term of A.P. be the first negative term.

i.e., $t_n < 0$

$a + (n-1)d < 0$

$20 + (n-1) \times \left(\frac{-3}{4}\right) < 0$

$(n-1)\left(-\frac{3}{4}\right) < -20$

$(n-1)\left(\frac{3}{4}\right) > 20$

$n - 1 > \frac{20}{\left(\frac{3}{4}\right)}$

$n - 1 > 20 \times \frac{4}{3}$

$n > \frac{80}{3} + 1$

$n > \frac{80+3}{3} = \frac{83}{3}$

$n > 27\frac{2}{3}$

$n \geq 28$ [$\because$ n is a natural number]

$\therefore$ If $n \geq 28$, the terms t_n becomes negative.

$\therefore 28^{\text{th}}$ term is the first negative term.

30) Find the sum of infinity of the G.P $\frac{-3}{4}, \frac{3}{16}, \frac{-3}{64}, \ldots$

Answer : Here $a=\frac{-3}{4}, r=\frac{-1}{4}$

Sum $=\frac{a}{1-r}=\frac{-\frac{3}{4}}{1-\left(\frac{-1}{4}\right)}$

Sum $=\frac{\frac{-3}{4}}{\frac{5}{4}}=\frac{-3}{5}$