QB365 Question Bank Software Study Materials

Trigonometry Important 2 Marks Questions With Answers (Book Back and Creative)

10th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Answer : $\tan^2\theta - \sin^2\theta = \tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}, \cos^2\theta$

$= \tan^2\theta (1-\cos^2\theta) = \tan^2\theta \sin^2\theta$

2) prove that $\frac{sinA}{1+cosA} = \frac{1-cosA}{sinA}$

Answer : $\frac{sinA}{1+cosA} = \frac{sinA}{1+cosA} \times \frac{1-cosA}{1-cosA}$ [multifly numerator and denominator by the conjugate of 1+cosA]

$= \frac{sinA(1-cosA)}{(1+cosA)\quad(1-cosA)} = \frac{sinA(1-cosA)}{1-cos^2A}$

$= \frac{sinA(1-cosA)}{sin^2A} = \frac{1-cosA}{sinA}$

3) prove that $1+\frac{cot^2\theta}{1+cosec\theta} = cosec\theta$

Answer : $1 + \frac{cot^2\theta}{1+cosec\theta+1} = 1+ \frac{cosec^2\theta-1}{cosec\theta+1}$ [since $cosec^2 - 1 = cot^2\theta$

$= 1+\frac{(cosec\theta+1)(cosec\theta-1)}{cosec\theta+1}$

$1 + (cosec\theta - 1) = cosec\theta$

4) prove that $\sec\theta - \cos\theta = \tan\theta \sin\theta$

Answer : $\sec\theta - \cos\theta = \frac{1}{cos\theta} - cos\theta = \frac{1-cos^2\theta}{cos\theta}$

$= \frac{sin^2\theta}{cos\theta}$ [since $1 - \cos^2\theta = \sin^2\theta$]

$= \frac{sin\theta}{cos\theta} \times sin\theta = tan\theta sin\theta$

5) prove that $\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = cot\theta$

Answer : $\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = \frac{\frac{1}{cos\theta}}{sin\theta} - \frac{sin\theta}{cos\theta} = \frac{1}{sin\theta cos\theta} - \frac{sin\theta}{cos\theta}$

$= \frac{1-sin^2\theta}{sin\theta cos\theta} = cot\theta$

6) prove the following identities. $\frac{1-tan^2\theta}{cot^2\theta-1} = tan^2\theta$

Answer : $\frac{1-\tan^2\theta}{\cot^2\theta-1} = \tan^2\theta$

$\text{LHS} = \frac{1-\tan^2\theta}{\cot^2\theta-1}$

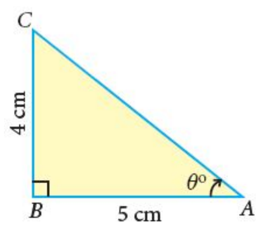
$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$

$= \frac{\frac{(\cos^2\theta-\sin^2\theta)}{\cos^2\theta}}{\frac{(\cos^2\theta-\sin^2\theta)}{\sin^2\theta}}$

$= \frac{(\cos^2\theta-\sin^2\theta)}{\cos^2\theta} \times \frac{\sin^2\theta}{(\cos^2\theta-\sin^2\theta)}$

$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \text{RHS}$

7) calculate ∠BAC in the given triangles (tan 38.7° = 0.8011)



Answer : in the right triangle ABC [see figure. (a)]

$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{4}{5}$

$= \tan^{-1}(0.8)$

θ = 38.7° (since tan 38.7° = 0.8011)

$\angle$BAC = 38.7°

8) A tower stands vertically on the ground. from a point on the ground,which is 48m away from the foot of the tower, the angel of elevation of the top of the tower is 30°.find the hieght of the tower.

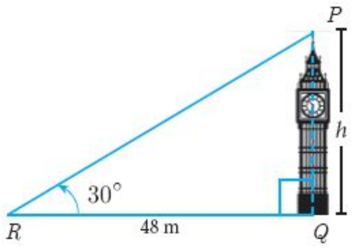
Answer : Let PQ the height of the tower.

Take PQ = h and QR is the distance between the tower and the point R.in right triangle PQR,$\angle$PRQ=30°

$\tan\theta = \frac{PQ}{QR}$

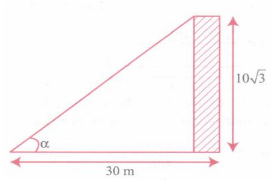
tan30° = $\frac{h}{48}$ gives, $\frac{1}{\sqrt{3}} = \frac{h}{48}$ so,h =$16\sqrt{3}$

Therefore the height of the tower $16\sqrt{3}$ m



9) Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}m$

Answer :



From the right $\triangle$ABC

$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AC}{BC}$

$= \frac{10\sqrt{3}m}{30m} = \frac{\sqrt{3}}{3}$

$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

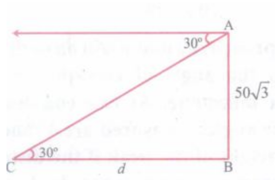
$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

0 = 30º

Angle of elevation is 30º

10) From the top of a rock $50\sqrt{3}$m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

Answer :



Let AB be the rock.

C be the position of the car.

o

In right triangle ABC

$\tan 30^{\circ} = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$

$BC = 50\sqrt{3} \times \sqrt{3}$

= 50 x 3 = 150 m

Distance of the car from the rock = 150 m.

11) A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball.($\sqrt{3}$ = 1.732)

Answer : Let BC be the height of the tower and A be the position of the ball lying on the ground. Then,

BC = 20 m and ∠XCA = 60° = ∠CAB

Let AB = x metres.

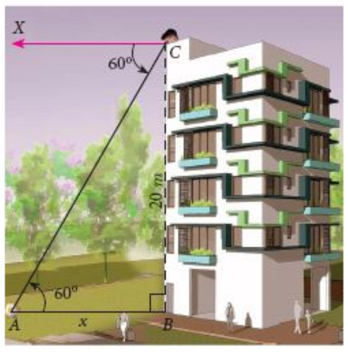
In the right angled ΔABC,

tan 60° = $\frac{BC}{AB}$

$\sqrt{3} = \frac{20}{x}$

$x = \frac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{20\times1.732}{3}$ = 11.54m

Hence, the distance between the foot of the tower and the ball is 11.55 m.



12) The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30° . If the height of the first building is 60 m, find the height of the second building.($\sqrt{3}$ = 1.732)

Answer : The height of the first building AB = 60 m. Now, AB = MD = 60 m

Let the height of the second building CD = h. Distance BD = 140 m

Now, AM = BD = 140 m

From the diagram,

∠XCA = 30° =∠CAM

In right triangle AMC, tan30° = $\frac{CM}{Am}$

$\frac{1}{\sqrt{3}} = \frac{CM}{140}$

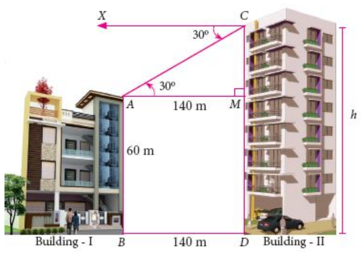
CM=$\frac{140}{\sqrt{3}} = \frac{140\sqrt{3}}{3}$

$= \frac{140\times1.732}{3}$

CM = 80.78

Now, h = CD = CM + MD = 80.78 + 60 = 140.78

Therefore the height of the second building is 140.78 m



13) prove the following identity.

$\frac{cos\theta}{1+sin\theta}$ = sec θ - tan θ

Answer : $\frac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

$\text{LHS} = \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

[Multiplying the Numerator and Denominator by 1 - sinθ]

$= \frac{\cos\theta(1-\sin\theta)}{1^2-\sin^2\theta}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$

$[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{(1-\sin\theta)}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$

= RHS

14) prove the following identities

$\sqrt{\frac{1+sin\theta}{1-sin\theta}} + \sqrt{\frac{1+sin\theta}{1-sin\theta}} = 2sec\theta$

Answer : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

$\text{LHS } =\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$

$=\sqrt{\frac{(1+\sin\theta)^2}{1^2-\sin^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$

$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$

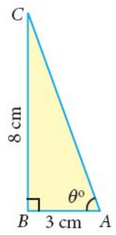
$=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$

$=2\times\frac{1}{\cos\theta}$

$=2\sec\theta$

= RHS

15) calculate $\angle$BAC inthe given triangles (tan 69.4° = 2.6604)



Answer : in right triangle ABC [see fig.(b)]

tanθ = $\frac{8}{3}$

= $\tan^{-1}$(2.66)

θ = 69.4°(since tan 69.4°=2.6604)

$\angle$BAC = 69.4°

16) When will the values of sin θ and cosθ be equal?

Answer : when θ = 45°

$\sin 45^\circ=\cos 45^\circ=\frac{1}{\sqrt{2}}$

$\therefore \sin\theta=\cos\theta \text{ for } \theta=45^\circ$

17) For what values of θ, sin θ = 2 ?

Answer : since $\sin\theta=\frac{\text{Opposite side}}{\text{Hypotenuse}}$,

it takes values From - 1 to 1.

For no real value, sin θ equal to 2.

18) Among the six trigonometric quantities, as the value of angle θ increase from 0° to 90° , which of the six trigonometric quantities has undefined values?

Answer : tan 90° is undefined

sec 90° is undefined

cosec 0° is undefined

cot 0° is undefined

19) Is it possible to have eight trigonometric ratios?

Answer : No. Since trigonometric ratios are relation between two of three sides of triangles only 6 combinations are there.

20) Let $0^\circ\leq\theta\leq 90^\circ$. For what values of θ does

sin θ > cos θ

Answer : $\sin 60^\circ=\frac{\sqrt{3}}{2};\cos 60^\circ=\frac{1}{2}$

$\Rightarrow\ \sin 60^\circ>\cos 60^\circ$

$\sin 90^\circ=1;$

$\Rightarrow\ \sin 90^\circ>\cos 90^\circ$

$\sin\theta>\cos\theta \text{ for } \theta=60^\circ \text{ and } \theta=90^\circ=0$

21) Let $0^\circ\leq\theta\leq 90^\circ$. For what values of θ does

cos θ > sin θ

Answer : $\sin 0^\circ = 0; \quad \cos 0^\circ = 1$

$\Rightarrow \quad \cos 0^\circ > \sin 0^\circ$

$\sin 30^\circ = \frac{1}{2}; \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos 30^\circ > \sin 30^\circ$

$\therefore \cos\theta > \sin\theta$ for $\theta = 0^\circ$ and $\theta = 30^\circ$

22) Let $0^\circ \le \theta \le 90^\circ$. For what values of θ does

sec θ = 2 tan θ

Answer : Given sec θ - 2tanθ

$\frac{\sec\theta}{\tan\theta} = 2$

$\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = 2$

$\frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2$

$\frac{1}{\sin\theta} = 2$

$\operatorname{cosec}\theta = 2$

$\therefore \operatorname{cosec}\theta = 2$ for $\theta = 30^\circ$

$\therefore \sec\theta = 2\tan\theta$ for $\theta = 30^\circ$

$Also \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

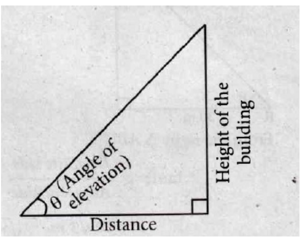
$2\tan 30^\circ = \frac{2}{\sqrt{3}} = \sec 30^\circ$

$\therefore \theta = 30^\circ$

23) What type of triangle is used to calculate heights and distances?

Answer : Right angled triangle is used to calculate heights and distances.

24) When the height of the building and distances from the foot of the building is given, which trigonometric ratio is used to find the angle of elevation?

Answer : 

If θ is the angle of elevation then the known measures are opposite side and adjacent side.

tan θ is used to find the angle of elevation.

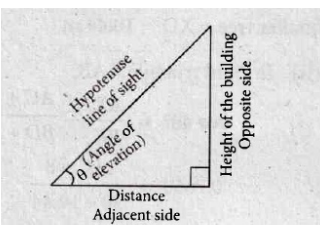
i.e., $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$= \frac{\text{Height of the building}}{\text{Distance}}$

25) If the line of sight and angle of elevation is given, then which trigonometric ratio is used

(i) to find the height of the building

(ii) to find the distance from the foot of the building.

Answer : 

(i) To find the height of the building

$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$ is used.

(ii) To find the distance from the foot of the building.

$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$ is used.

26) (sin ∝+cos ∝)(tan ∝+cot ∝)=sec ∝+cosec ∝

Answer :

27) tan θ+tan(90°-θ)=secθ sec(90°-θ)

Answer :

28) If $2\sin^2\theta - \cos 2\theta = 2$, then find the value θ.

Answer :

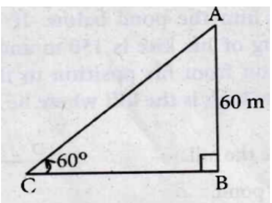
29) Prove that trigonometrical identity $\cos^2\theta\left(1+\tan^2\theta\right)=1$

Answer : $\text{LHS} = \cos^2\theta\left(1+\tan^2\theta\right)$

$= \cos^2\theta \sec^2\theta \quad \left[\because 1+\tan^2\theta=\sec^2\theta\right]$

$= \cos^2\theta \times \frac{1}{\cos^2\theta} = 1 = \text{RHS}$

30) A kite is flying at a height of 60 m above the ground. The inclination of the string with the ground where its string is tied is 60°. Find the length of the string.

Answer : 

Let AB be the height of the kite from the ground'

AC is the length of the string.

From the right triangle ΔABC

$\sin 60^\circ = \frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{60}{AC}$

$\text{AC} = \frac{2\times 60}{\sqrt{3}}$

$= \frac{2\times 20\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$= 40\sqrt{3}m$

$\text{Length of the string } = 40\sqrt{3}\text{ m}$