

Answer : Let $P(x) = x^3 + 5x^2 - 10x + 4$

By factor theorem $(x-1)$ is a factor of $P(x)$, if $P(1) = 0$

$$P(1) = 1^3 + 5(1^2) - 10(1) + 4$$

$$= 1 + 5 - 10 + 4$$

$$P(1) = 0$$

$\therefore (x-1)$ is a factor of $x^3 + 5x^2 - 10x + 4$

- 10) Expand $(5a - 3b)^3$

Answer : We know that, $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$$(5a - 3b)^3 = (5a)^3 - (5a)^2(3b) + 3(5a)(3b)^2 - (3b)^3$$

$$= 125a^3 - 3(25a^2)(3b) + 3(5a)(9b^2) - (3b)^3$$

$$= 125a^3 - 225a^2b + 135ab^2 - 27b^3$$

The following identity is also used:

$$x^3 + y^3 + z^3 - 3xyz \equiv (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

We can check this by performing the multiplication on the right hand side.

- 11) Expand: $\left(x + \frac{1}{y}\right)^3$

Answer : $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$$\left(x + \frac{1}{y}\right)^3 = x^3 + \frac{3x^2}{y} + \frac{3x}{y^2} + \frac{1}{y^3}$$

- 12) Find $27a^3 + 64b^3$, if $3a + 4b = 10$ and $ab = 2$.

Answer : $3a+4b = 10$, $ab = 2$

$$(3a + 4b)^3 = (3a)^3 + 3(3a)^2(4b)^2 + (4b)^3$$

$$\therefore x^3 + y^3 = (x+y)^3 - 3xy - (x+y)$$

$$= 10^3 - 36ab(10) = 1000 - 36 \times 2 \times 10$$

$$= 1000 - 720 = 280$$

- 13) If $x^2 + \frac{1}{x^2} = 23$, then find the value of $x + \frac{1}{x}$ and $x^3 + \frac{1}{x^3}$.

Answer : $(a+b)^2 = a^2 + b^2 + 2ab$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2x\frac{1}{x}$$

$$= 23 + 2 = 25$$

$$\therefore x + \frac{1}{x} = \pm 5$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)(x + \frac{1}{x})$$

$$5^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$125 = x^3 + \frac{1}{x^3} + 3(5)$$

$$125 - 15 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = \pm 110$$

- 14) Factorise the following: $9x^2 + 12xy + 4y^2$

Answer : $9x^2 + 12xy + 4y^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$ [$a^2 + 2ab + b^2 = (a+b)^2$]

$$= (3x + 2y)^2$$

- 15) Factorise the following: $27x^3 + 125y^3$

Answer : $27x^3 + 125y^3 = (3x)^3 + (5y)^3$ [$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$]

$$= (3x + 5y)((3x^2) - (3x)(5y) + (5y)^2)$$

$$= (3x + 5y)(9x^2 - 15xy + 25y^2)$$

- 16) Factorise the following: $216m^3 - 343n^3$

Answer : $216m^3 - 343n^3 = (6m)^3 - (7n)^3$ [$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$$= (6m - 7n)((6m)^2 + (6m)(7n) + (7n)^2)$$

$$= (6m - 7n)(36m^2 + 42mn + 49n^2)$$

- 17) Factorise the following: $8x^3 + 27y^3 + 64z^3 - 72xyz$

Answer : $8x^3 + 27y^3 + 64z^3 - 72xyz$
 $= (2x)^3 + (3y)^3 + (4z)^3 - 3(2x)(3y)(4z)$
 $= (2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 16xy - 12yz - 8xz)$

- 18) Factorise the following: $a^6 - 64$

Answer : $(a^2)^3 - 4^3$
 $= (a^2 - 4)(a^4 + 4a^2 + 4^2)$
 $= (a+2)(a-2)(a^2+4-2a)(a^2-4+2a)$

- 19) Factorise the following: $x^3 + 8y^3 + 6xy - 1$

Answer : $x^3 + (2y)^3 + (-1)^3 - 3(x)(2y)(-1)$
 $= (x+2y-1)(x^2 + 4y^2 + 1 - 2xy + 2y+x)$

- 20) ABCD is a cyclic quadrilateral such that $\angle A = (4y + 20)^\circ$, $\angle B = (3y - 5)^\circ$, $\angle C = (4x)^\circ$ and $\angle D = (7x + 5)^\circ$. Find the four angles.

Answer : $\angle A + \angle C = 180^\circ$

$$4y + 20 + 4x = 180$$

$$4x + 4y = 180 - 20$$

$$x + y = \frac{160}{4}$$

$$x + y = 40 \quad \dots(1)$$

$$\angle B + \angle D = 180^\circ$$

$$3y - 5 + 7x + 5 = 180$$

$$7x + 3y = 180 \quad \dots(2)$$

$$(1) \times 7 \Rightarrow 7x + 7y = 280$$

$$(2) \Rightarrow \frac{7x + 3y = 180}{4y = 100} \quad \dots(2)$$

$$y = 25^\circ$$

Substitute $y = 25^\circ$ in (1)

$$x + 25 = 40$$

$$x = 40 - 25$$

$$x = 15^\circ$$

$$\angle A = (4y + 20)^\circ = (4 \times 25 + 20) = 100 + 20 = 120^\circ$$

$$\angle B = (3y - 5)^\circ = (3 \times 25^\circ - 5) = 75^\circ - 5^\circ = 70^\circ$$

$$\angle C = (4x)^\circ = (4 \times 15^\circ) = 60^\circ$$

$$\angle D = (7x + 5)^\circ = (7 \times 15 + 5) = 105^\circ + 5^\circ = 110^\circ$$

The Four angles are $\angle A = 120^\circ$, $\angle B = 70^\circ$, $\angle C = 60^\circ$, $\angle D = 110^\circ$

- 21) Simplify $3y(2y - 7) - 3(y - 4) - 63$ and evaluate when $y = -2$.

Answer : $x^2 - 3x + 2$.

The value is 0

- 22) Factorise the following: $a^3 + 3a^2b + 3ab^2 + 2b^3$

Answer : $a^3 + 3a^2b + 3ab^2 + 2b^3$
 $= (a+2b)(a^2+b^2+ab)$

- 23) Factorise the following : $x^4 - 9x^2$

Answer : $x^4 - 9x^2 = x^2(x^2 - 9) = x^2(x^2 - 3^2) = x^2(x - 3)(x + 3)$

- 24) Draw the graph for the following linear equations

(i) $y = 4$

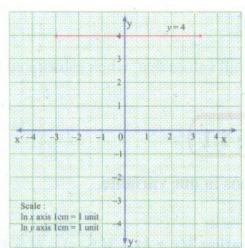
(ii) $x = -2$

(iii) $2x - 4 = 0$

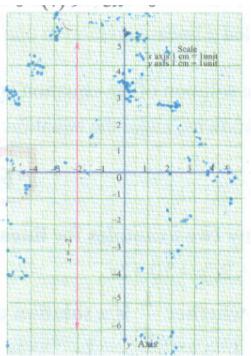
(iv) $6 + 2y = 0$

(v) $9 - 3x = 0$

Answer : (i)



(ii)

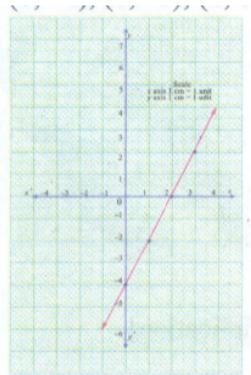


(iii)

x	0	1	2	3
2x	0	2	4	6
-4	-4	-4	-4	-4
y = 2x - 4	-4	-2	0	2

\therefore The points to be plotted:

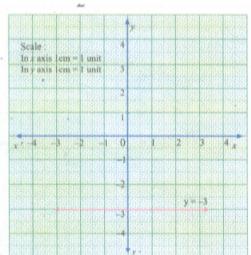
$$(0, -4), (1, -2), (2, 0), (3, 2)$$



(iv)

$$2y = -6$$

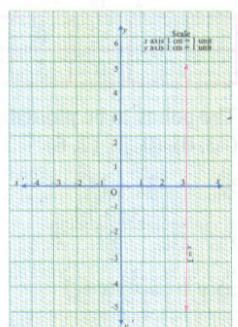
$$y = \frac{-6}{2} \Rightarrow y = 3$$



$$(v) -9 + 9 - 3x = 0 -9$$

$$\cancel{-3x} = \cancel{-9}$$

$$x = \frac{9}{3} \Rightarrow x = 3$$



25) Solve by the method of elimination.

$$2x + y = 10; 5x - y = 11$$

Answer : $2x + y = 10$ ----- (1)

$$5x - y = 11 \text{ ----- (2)}$$

$$(1)+(2) \Rightarrow$$

$$2x + y = 10$$

$$\begin{array}{r} 5x - y = 11 \\ 7x = 21 \end{array}$$

$$x = \frac{21}{7} = 3$$

$$(1) \Rightarrow 2(3) + y = 10$$

$$6 + y = 10$$

$$y = 10 - 6 = 4$$

\therefore Solution is $x = 3, y = 4$

- 26) Solve by cross multiplication method.

$$37a + 29b = 45; 29a + 37b = 21$$

Answer : $37a + 29b - 45 = 0$

$$29a + 37b - 21 = 0$$

$$\begin{array}{c} 29 \quad a \quad -45 \quad b \quad 37 \quad 1 \quad 29 \\ \times \quad \quad \quad \quad \times \quad \quad \quad \times \\ 37 \quad -21 \quad 29 \quad 37 \end{array}$$

$$\frac{a}{29(-21) - 37(-45)} = \frac{b}{(-45)(29) - (-21)37} = \frac{1}{37(37) - 29(29)}$$

$$\frac{a}{-609 + 1665} = \frac{b}{-1305 + 777} = \frac{1}{1369 - 841}$$

$$\frac{a}{1056} = \frac{b}{-528} = \frac{1}{528}$$

$$\frac{a}{1056} = \frac{b}{-528} \Rightarrow a \cdot \frac{1056}{528} = 2$$

$$\frac{b}{-528} = \frac{1}{528} \Rightarrow b = \frac{-528}{528} = -1$$

- 27) Expand

i) $(3a + 4b)^3$

ii) $(2x - 3y)^3$

Answer : i) $(3a + 4b)^3 = (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3$

$$= 27a^3 + 108a^2b + 144ab^2 + 64b^3$$

ii) $(2x - 3y)^3 = (2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^3$

$$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$$

- 28) Factorize:

(i) $3x^2 + 19x + 6$

(ii) $5y^2 + 29 + 20$

(iii) $18x^2 - x - 4$

(iv) $2x^2 - 3x - 14$

Answer : i) $3x^2 + 19x + 6 = 3x^2 + 18x + x + 6$

$$= 3x(x + 6) + 1(x + 6)$$

$$= (3x + 1)(x + 6)$$

ii) $5y^2 + 29 + 20 = 5y^2 - 25y - 4y + 20$

$$= 5y(y - 5) - 4(y - 5)$$

$$= (5y - 4)(y - 5)$$

iii) $18x^2 - x - 4 = 18x^2 - 9x + 8x - 4$

$$= 9x(2x - 1) + 4(2x - 1)$$

$$= (9x + 4)(2x - 1)$$

iv) $2x^2 - 3x - 14 = 2x^2 - 7x + 4x - 14$

$$= 2x(2x - 7) + 2(2x - 7)$$

$$= (2x - 7)(x + 2)$$

- 29) If the polynomials $x^3 + 3x^2 - m$ and $2x^2 - mx + 9$ leaves the same remainder when they are divided by $(x-2)$. Find the value of m . Also find the remainder.

Answer : Let $p(x) = x^3 + 3x^2 - m$ and $q(x) = 2x^2 - mx + 9$

When $p(x)$ and $q(x)$ are divided by $(x - 2)$, we get same remainder

i.e, $p(2) = q(2)$

$$2^3 + 3(2)^2 - m = 2(2)^3 - m(2) + 9$$

$$8 + 12 - m = 16 - 2m + 9$$

$$20 - m = 25 - 2m$$

$$2m - m = 25 - 20$$

$$m = 5$$

Reminder:

$$= P(2)$$

$$= 2^3 + 3(2)^2 - 5$$

$$= 20 - 5$$

$$= 15$$

30) $4x^3 - 11x^2 - 7x + 3?$

Answer : Let $p(x) = 4x^3 - 11x^2 - 7x + 3$

$$p(1) = 4(1)^3 - 7(1) + 3$$

$$= 7 - 7 = 0$$

$(x - 1)$ is a factor of $p(x)$

$$\begin{array}{r} 4 \quad 0 \quad -7 \quad 3 \\ 1 | 0 \quad 4 \quad 4 \quad -3 \\ \hline 4 \quad 4 \quad -3 \quad 0 \end{array} \rightarrow \text{Remainder}$$

The other factor is $4x^2 + 4x - 3$

$$4x^2 + 4x - 3 = (2x + 3)(2x - 1)$$

$$\therefore 4x^3 - 11x^2 - 7x + 3 = (2x + 3)(2x - 1)(x - 1)$$